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Tables of Fourier, Laplace and Hankel transforms of n-dimensional generalized functions

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by

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1994

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Introduction

Let T be a distribution. By λT we designate the distribution defined by the scalar product

$$< \lambda T, \phi > = < T, \lambda^{-1} \phi >,$$

where λ is a rotation.

We say T is rotation-invariant if

$$\lambda T = T,$$

for every λ.

By $S'^{\natural}_{R^n}$ we designate the space of tempered rotation-invariant distributions defined in R^n.

Let now $\tilde{T}$ be the image of $T \in S'^{\natural}_{R^n}$ in S'_{R+}; one has ([30], eq (AI,2,3))

$$< \tilde{T}, \phi(t) > = < T, \phi(r^2) >,$$

for every $\phi \in S_{R+}$.

We have (cf.[3])

Theorem. (The causal version of the distributional Bochner Theorem)

Let $f(z, \lambda)$ be an entire function in the variables z, λ and let $T(P \pm i0, \lambda) = (P \pm i0)^\lambda f(P \pm i0, \lambda) \in S'$.

Then, the following formula is valid:

$$\{T(P \pm i0, \lambda)\}^\wedge = e^{\mp i \frac{\pi}{2} q} (\mathcal{H}\{\tilde{T}(|x|^2, \lambda)\})_{|y|^2 \to Q \mp i0}.$$

The equivalence between the Bochner formula and the Hankel transform

Let $f(x)$ be an integrable and rotation-invariant function belongs to R^n. We suppose that $f(x)$ further, satisfies $f(x) \in C_0(R^n)$ and $\mathcal{F}[f(y)] = \Psi(y) = \varphi(\rho^2) \in L(R^n)$.

We define

$$\hat{f}(x) = \frac{1}{(2\pi)^n} \int_{R^n} \Psi(y) e^{i<x,y>} dy.$$

We have, by means of the Bochner formula,

$$f(x) = g(r^2) = \frac{1}{(2\pi)^n} \frac{(2\pi)^{\frac{n}{2}}}{r^{\frac{n-2}{2}}} \int_0^\infty \varphi(t^2) t^{\frac{n}{2}} J_{\frac{n-2}{2}}(rt) dt.$$

Therefore, we have the following pair of reciprocal formulas:

$$\varphi(\rho^2) = \frac{(2\pi)^{\frac{n}{2}}}{\rho^{\frac{n-2}{2}}} \int_0^\infty g(t^2) t^{\frac{n}{2}} J_{\frac{n-2}{2}}(\rho t) dt,$$

and

$$g(r^2) = \frac{1}{(2\pi)^{\frac{n}{2}}} \frac{1}{r^{\frac{n-2}{2}}} \int_0^\infty \varphi(t^2) t^{\frac{n}{2}} J_{\frac{n-2}{2}}(rt) dt.$$

Finally, we can enunciate the following

Theorem:

Hypothesis: Let $g(t)$ be a continuous function in $[0, \infty)$,

$$\int_0^\infty |g(t^2)| t^{n-1} dt < \infty,$$

$$\int_0^\infty |\varphi(\rho^2)| \rho^{n-1} d\rho < \infty.$$

Thesis: The following formulas are valid:

$$\varphi(\rho^2) = \frac{(2\pi)^{\frac{n}{2}}}{\rho^{\frac{n-2}{2}}} \int_0^\infty g(t^2) t^{\frac{n}{2}} J_{\frac{n-2}{2}}(\rho t) dt,$$

$$g(r^2) = \frac{1}{(2\pi)^{\frac{n}{2}}} \frac{1}{r^{\frac{n-2}{2}}} \int_0^\infty \varphi(t^2) t^{\frac{n}{2}} J_{\frac{n-2}{2}}(rt) dt.$$

The above formulae are a particular case of the Hankel formulas.

Let $G_\alpha(P \pm i0, m, n)$ be the causal (anticausal) distribution defined by

$$G_\alpha(P \pm i0, m, n) = H_\alpha(m, n)(P \pm i0)^{\frac{1}{2}(\frac{\alpha-n}{2})} K_{\frac{n-\alpha}{2}}(\sqrt{m^2(P \pm i0)}), \qquad (A,25)$$

where m is a positive real number, $\alpha \in C$, K_μ designates the well-known modified Bessel function of the third kind (cf. [Watson], p. 78, formulas (6) and (7)):

$$K_\nu(z) = \frac{\pi}{2} \frac{I_{-\nu} - I_\nu(z)}{\sin(\pi\nu)}, \qquad (A,26)$$

$$I_\nu(z) = \sum_{m=0}^{\infty} \frac{(\frac{z}{2})^{2m+\nu}}{m!\Gamma(m+\nu+1)}, \qquad (A,27)$$

$$H_\alpha(m, n) = \frac{e^{1\frac{\pi}{2}\alpha} 2^{1-\frac{\alpha}{2}} e^{\pm i\frac{\pi}{2}q} (m^2)^{\frac{1}{2}(\frac{n-\alpha}{2})}}{(2\pi)^{\frac{n}{2}} \Gamma(\frac{\alpha}{2})}. \qquad (A,28)$$

We observe that the distributional function $G_\alpha(P \pm i0, m)$ is a (causal, anticausal) analogue of the kernel due to A.P. Calderón, Aronszajn-Smith and L. Schwartz (cf. [9], [10] and [4], respectively.

The distributions $G_\alpha(P \pm i0, m, n)$ share many properties with the Bessel kernel of which they are (causal, anticausal) analogues (cf. [3]).

Let us define the n-dimensional ultrahyperbolic Klein-Gordon operator iterated l-times

$$K^l = \{\frac{\partial^2}{\partial x_1^2} + \cdots + \frac{\partial^2}{\partial x_p^2} - \frac{\partial^2}{\partial x_{p+1}^2} - \cdots - \frac{\partial^2}{\partial x_{p+q}^2} - m^2\}^l, \qquad (A,30)$$

where $p + q = n$ and $m > 0$.

We define the causal (anticausal) distribution $H_\alpha(P \pm i0, n)$ as follows:

$$H_\alpha(P \pm i0, n) = \frac{e^{i\frac{\pi}{2}\alpha} e^{\pm i\frac{\pi}{2}q} \Gamma(\frac{n-\alpha}{2})}{2^\alpha \pi^{\frac{n}{2}} \Gamma(\frac{\alpha}{2})} (P \pm i0)^{\frac{\alpha-n}{2}}, \qquad (A,31)$$

where $\alpha \in C$, P is defined by (A,8) and q is the number of negative terms of the quadratic form P. The distributional functions H_α are the causal (anticausal) analogues of the elliptic kernel (cf. [36], pp. 16-21) and have analogue properties, that we use to obtain causal (anticausal) solutions of the n-dimensional ultrahyperbolic operator, iterated k-times (k integer ≥ 1):

$$L^k = \{\frac{\partial^2}{\partial x_1^2} + \cdots + \frac{\partial^2}{\partial x_p^2} - \frac{\partial^2}{\partial x_{p+1}^2} - \cdots - \frac{\partial^2}{\partial x_{p+q}^2}\}^k, \quad p + q = n. \qquad (A,32)$$

By putting $q = 0, p = n$ in the definitory formula of $H_\alpha(P \pm i0, n)$ (cf. (A,31), we obtain

$$H_\alpha(|x|^2, n) \stackrel{\text{def}}{=} R_\alpha(x,n) = \frac{r^{\alpha-n}}{D_n(\alpha)}, \qquad (A,33)$$

where

$$r^2 = |x|^2 = x_1^2 + \cdots + x_n^2, \qquad (A,34)$$

and

$$D_n(\alpha) = \frac{\pi^{\frac{n}{2}} 2^\alpha \Gamma(\frac{\alpha}{2})}{\Gamma(\frac{n-\alpha}{2})}. \qquad (A,35)$$

Formula (A,33) is, precisely, the definition of the elliptic kernel of M. Riesz (cf [10]).

Let us define the n-dimensional Laplacian operator iterated s-times:

$$\Delta^s = \{\frac{\partial^2}{\partial x_1^2} + \cdots + \frac{\partial^2}{\partial x_n^2}\}^s. \qquad (A,36)$$

We begin by observing that the function H_α has simple poles at $\alpha = n + 2l$, $l =$ integer ≥ 0 (which are due to the Γ which appears in the numerator). This is an essential difference between H_α and G_α, which is an entire distribution.

H_α admits a Laurent expansion in the neighbourhood of the point $\alpha = n + 2r$, $r = 0, 1, \ldots$:

$$H_\alpha(P \pm i0, n) = \frac{A-1}{\alpha - (n+2r)} + A_0 + \sum_{\nu=1}^{\infty} A_\nu(\alpha - (n+2r))^\nu \qquad (A,37)$$

The distribution A_0 is, by definition, the <u>finite part</u> of $H_\alpha(P \pm i0, n)$ at

$$\lambda = n + 2r, \qquad r = 0, 1, \ldots. \qquad (A,38)$$

The Bessel function of the first kind is defined by (cf. [11], p. 4, formula (2))

$$J_\nu(z) = \sum_{m=0}^{\infty} \frac{(-1)^m (\frac{z}{2})^{\nu+2m}}{m!\Gamma(m+\nu+1)}. \qquad (A,39)$$

The Bessel function of the second kind or Neumann's function $Y_\nu(z)$ is given by (cf. [11], p. 4, formula, (4)):

$$Y_\nu(z) = (\sin \nu\pi)^{-1}[J_\nu(z)\cos(\nu\pi) - J_{-\nu}(z)]. \qquad (A,40)$$

The linear combinations

$$H_\nu^{(1)}(z) = J_\nu(z) + iY_\nu(z) = [i \sin(\nu\pi)]^{-1}[J_{-\nu}(z) - J_\nu(z)e^{-i\nu\pi}], \qquad (A,41)$$

and

$$H_\nu^{(2)}(z) = J_\nu(z) - iY_\nu(z) = (i \sin \nu\pi)^{-1}[J_\nu(z)e^{i\nu\pi} - J_{-\nu}(z)]. \qquad (A,42)$$

are called the Bessel functions of the third kind or the first and second Hankel functions. We also can write

$$K_\nu(z) = \frac{1}{2} i\pi e^{i\frac{1}{2}\nu\pi} H_\nu^{(1)}(ze^{i\frac{\pi}{2}}), \qquad (A,43)$$

and

$$K_\nu(z) = -\frac{1}{2} i\pi e^{-i\frac{1}{2}\nu\pi} H_\nu^{(2)}(ze^{-i\frac{\pi}{2}}). \qquad (A,44)$$

We begin this paragraph with several definitions.

Let $t = (t_0, t_1, \ldots, t_{n-1})$ be a point of R^n. We shall write $t_0^2 - t_1^2 - \cdots - t_{n-1}^2 = u$. By Γ_+ we designate the interior of the forward cone :

$$\Gamma_+ = \{t \in R^n / t_0 > 0, u > 0\} \qquad (A,45)$$

and by $\overline{\Gamma}_+$ we designate its closure. Similarly, Γ_- designates the domain

$$\Gamma_- = \{t \in R^n / t_0 < 0, u > 0\} \qquad (A,46)$$

and $\overline{\Gamma}_-$ designates its closure. We put $z = (z_0, z_1, \ldots, z_{n-1}) \in C^n$, where $z_\nu = x_\nu + iy_\nu, \nu = 0, 1, \ldots, n-1$; $< t, z >= t_0 z_0 + t_1 z_1 + \cdots t_{n-1} z_{n-1}$; and $dt = dt_0 dt_1 \ldots dt_{n-1}$.

The tube T_- is defined by

$$T_- = \{z \in C^n / y \in V_-\},$$

where $\nu_- = \{y \in R^n \ / \ y_0 < 0, y_0^2 - y_1^2 \cdots - y_{n-1}^2 > 0\}$. Similarly, we put the tube

$$T_+ = \{z \in C^n / y \in \nu_+\},$$

where $\nu_+ = \{y \in R^n / y_0 > 0, y_0^2 - y_1^2 \cdots - y_{n-1}^2 > 0\}$.

Let $F(\lambda)$ be a function of the scalar variable λ, and let $\phi(t)$ be a function endowed with the following properties:

a) $\phi(t) = F(u)$,

b) supp $\phi(t) \subset \overline{\Gamma}_+$,

c) $e^{<t,y>}\phi(t) \in L_1$, if $y \in \nu_-$.

We call $\mathcal{R}$ the family of functions $\phi(t)$ which satisfies a), b) and c). Similarly, we call $\mathcal{A}$ the family of functions which satisfies conditions

a') $\phi(t) = F(u)$,

b') supp $\phi(t) \in \overline{\Gamma}_-$,
c') $e^{<t,y>}\phi(t) \in L_1$, if $y \in \nu_+$.

Let $\phi(t)(t = t_1, t_2, \ldots, t_n)$ be a radial integrable function: $\phi(t) = F(r^2)$, where $r^2 = \sum_{i=1}^n t_i^2$ and $F(\lambda)$ is a function of the scalar variable λ.

Let $F[\phi]$ designate the Fourier transform of ϕ:

$$F[\phi] = \int_{R^n} e^{-i<t,x>}\phi(t)dt.$$

A classical Bochner formula ([16], p. 187) expresses $F[\phi]$ by means of a Hankel transform:

$$F[\phi] = \frac{(2\pi)^{\frac{n}{2}}}{\{x_0^2 + x_1^2 + \cdots + x_{n-1}^2\}^{\frac{n-2}{4}}} \cdot \int_0^\infty F(\lambda)\lambda^{\frac{n}{2}} J_{\frac{n-2}{2}}\{\lambda(x_0^2 + \cdots + x_{n-1}^2)^{\frac{1}{2}}\}d\lambda.$$

Here $J_\nu(z)$ is the well-known Bessel function:

$$J_\nu(z) = \sum_{p=0}^\infty \frac{(-1)^p(\frac{z}{2})^{\nu+2p}}{p!\Gamma(p+\nu+1)}.$$

Theorem. (Bochner [16], p. 187)

Hypothesis:
a) $f(x) \in L(R^n)$,
b) $f(x) = g(r^2)$,
c) $f(x) = f(-x)$, if $n = 1$.

Thesis:

$$F[f] = \phi(\rho^2) = \frac{(2\pi)^{\frac{n}{2}}}{\rho^{\frac{n-2}{2}}} \int_0^\infty g(t^2)t^{\frac{n}{2}} J_{\frac{n-2}{2}}(\rho t)dt.$$

Here $J_\nu(z)$ is the well-known Bessel function

$$J_\nu(z) = \sum_{p=0}^\infty (-1)^p \frac{(\frac{z}{2})^{\nu+2p}}{p!\Gamma(p+\nu+1)}.$$

The following assertion proves an analog of Bochner 's formula for Laplace transforms of the form

$$f(z) = L[\phi] = \int_{R^n} e^{-i<t,z>}\phi(t)dt,$$

where ϕ is a function of the Lorentz distance, whose support is contained in the closure of the domain $t_0 > 0, t_0^2 - t_1^2 - \cdots - t_{n-1}^2 > 0$.

The Laplace transform of $\phi(t)$ is $f(z) = L[\phi] = \int_{R^n} e^{-i<t,z>}\phi(t)dt$. We can state the following

Theorem.

Hypothesis:

a) $\phi(t) \in \mathcal{R}$,

b) $z \in T_-$.

Thesis:

$$f(z) = L\{\phi\} = \frac{(2\pi)^{\frac{n-2}{2}}}{\{z_1^2 + \cdots + z_{n-1}^2 - z_0^2\}^{\frac{n-2}{2}}} \cdot \int_0^\infty F(\lambda)\lambda^{\frac{n-2}{4}} K_{\frac{n-2}{2}}$$
$$\{(\lambda\sqrt{z_1^2 + \cdots + z_{n-1}^2 - z_0^2})\}dx.$$

Here $K_\nu(z)$ designates the modified Bessel function of the third kind (cf. [11]).

We also have the two following representation formulae:

Theorem.

Hypothesis: The same as that of above Theorem.

Thesis.
(a) If $n = 2m + 2, m = 0, 1, \ldots,$

$$f(z) = L[\phi] = (-1)^m 2^{2m}\pi^m \frac{d^m}{ds^m}\int_0^\infty F(\lambda)K_0[(\lambda s)^{\frac{1}{2}}]d\lambda.$$

(b) If $n = 2m + 1, m = 1, 2, \ldots,$

$$f(z) = L[\phi] = (-1)^m 2^{2m-1}\pi^m \frac{d^m}{ds^m}\int_0^\infty F(\lambda)\lambda^{-\frac{1}{2}}e^{-(\lambda s)^{\frac{1}{2}}}d\lambda.$$

Notes:

1. Formula (b) seems to be equivalent to a very interesting result due to Leray ([15], p.41, formulas (19,11)), which he proves by a completely different method.

2. Formulas a) and b) are analogs, for Laplace transforms, of two Bochner formulas ([16], p. 187, formulas (17) and (18)), valid for Fourier transforms of radial functions.

We define the following functions of the family $\mathcal{R}$:

$$G_{\mathcal{R}}(t,\alpha,m^2,n)=\frac{(u-m^2)_+^{\alpha-1}}{\Gamma(\alpha)}=\begin{cases}\frac{(u-m^2)^{\alpha-1}}{\Gamma(\alpha)} & \text{if } u-m^2>0 \text{ and } t>0,\\ 0 & \text{if } t \text{ belongs to the complementary set.}\end{cases} \tag{A,47}$$

$$G_{\mathcal{R}}(t,\alpha=1,m^2,n)=\begin{cases}1 & \text{if } u-m^2>0 \text{ and } t_0>0,\\ 0 & \text{if } t \text{ belongs to the complementary set.}\end{cases} \tag{A,48}$$

$$G_{\mathcal{R}}(t,\alpha=-k,m^2,n)=\delta_{\mathcal{R}}^{(k)}(u-m^2). \tag{A,49}$$

$$G_{\mathcal{R}}(t,\alpha,m=0,n)=\frac{u_+^{\alpha-1}}{\Gamma(\alpha)}=\begin{cases}\frac{u^{\alpha-1}}{\Gamma(\alpha)} & \text{if } u>0 \text{ and } t_0>0,\\ 0 & \text{if } t \text{ belongs to the complementary set.}\end{cases} \tag{A,50}$$

The characteristic function of the volume bounded by the forward cone:

$$G_{\mathcal{R}}(t,\alpha=1,m=0,n)=\begin{cases}1 & \text{if } u>0, t_0>0,\\ 0 & \text{if } t \text{ belongs to the complementary set.}\end{cases} \tag{A,51}$$

$$G_{\mathcal{R}}(t,\alpha=-k,m=0,n)=\delta_{\mathcal{R}}^{(k)}(u). \tag{A,52}$$

We now define the following n-dimensional function of the family $\mathcal{A}$:

$$G_{\mathcal{A}}(t,\alpha,m^2,n)=\frac{(u-m^2)_-^{\alpha-1}}{\Gamma(\alpha)}=\begin{cases}\frac{(u-m^2)^{\alpha-1}}{\Gamma(\alpha)} & \text{if } u-m^2>0 \text{ and } t_0<0,\\ 0 & \text{if } t \text{ belongs to the complementary set.}\end{cases} \tag{A,53}$$

Finally, we introduce the function $G(t,\alpha,m^2,n)$ defined by

$$G(t,\alpha,m^2,n)=G_{\mathcal{R}}(t,\alpha,m^2,n)+G_{\mathcal{A}}(t,\alpha,m^2,n). \tag{A,54}$$

We shall consider the following functions of the family $\mathcal{R}$ introduced by M. Riesz (cf. [10], p. 89, [4], p. 179).

$$W(t,\alpha,m^2,n)=\begin{cases}\frac{(m^{-2}u)^{\frac{\alpha-n}{4}}}{\pi^{\frac{n-2}{2}}2^{\frac{2\alpha+n-2}{2}}\Gamma(\frac{\alpha}{2})}J_{\frac{\alpha-n}{2}}(\sqrt{m^2u}) & \text{if } t\in\Gamma_+,\\ 0 & \text{if } t\notin\Gamma_+.\end{cases} \tag{A,55}$$

Here α is a complex parameter, m a real nonnegative number and n the dimension of the space.

$W(t,\alpha,m^2,n)$, which is an ordinary function if $Re\alpha \geq n$ is an entire distributional function of α.

Putting $m=0$ in (A,55), we obtain (cf. [12], formula (II,3,1), p.11)

$$W(t,\alpha,m=0,n) = R_\alpha(u) = \begin{cases} \frac{u^{\frac{\alpha-n}{2}}}{H_n(\alpha)} & \text{if } t \in \Gamma_+, \\ 0 & \text{if } t \notin \Gamma_+. \end{cases} \qquad (A,56)$$

Here we have put

$$H_n(\alpha) = \pi^{\frac{n-2}{2}} 2^{\alpha-1} \Gamma(\frac{\alpha}{2}) \Gamma(\frac{\alpha-n+2}{2}). \qquad (A,57)$$

We remember that

$$\Psi(z) = \frac{\Gamma'(z)}{\Gamma(z)}, \qquad (A,58)$$

and

$$\Omega_n = \frac{2\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2})}. \qquad (A,59)$$

Let $\Phi(t)$ be defined in $R^+ : \{t, t>0\}$. By the Hankel transform of the function $\Phi(t)$ we can mean the function $g(s), 0 \leq s < \infty$, defined by the formula

$$g(s) = (\mathcal{H}\{\phi(t)\}) = \frac{1}{2}\int_0^\infty \phi(t) t^{\frac{n-2}{2}} R_{\frac{n-2}{2}}(\sqrt{st})dt,$$

where

$$R_m(x) = \frac{J_m(x)}{x^m},$$

and $J_m(x)$ is the well-known Bessel function defined by the formula

$$J_m(x) = \sum_{\nu=0}^{\infty} \frac{(-1)^\nu}{\nu!} \frac{(\frac{\nu}{2})^{m+2\nu}}{(m+\nu+1)}.$$

It is well-known (cf. [33], p.240) that if $\phi(t)$ satisfies adequate conditions, for example, if $\phi(t) \in S_{R^+}$, the following formula is valid:

$$\phi(t) = (\mathcal{H}\{g(s)\}) = \frac{1}{2}\int_0^\infty g(s) s^{\frac{n-2}{2}} R_{\frac{n-2}{2}}(\sqrt{st})ds.$$

Let S_{R^+} designate the space of functions $f \in S$ defined in the positive half line $R^+ : \{t, t>0\}$. By S'_{R^+} we designate the dual of S_{R^+}.

Let $U(t) \in S'_{R+}$. The Hankel transform of $U(t)$ will be, by definition, the distribution $\nu(s) \in S'_{R+}$ defined by the formula

$$< \mathcal{H}(U(t)), \phi(s) >=< U(t), (\mathcal{H}\{\phi(s)\}) >,$$

for every $\phi \in S_{R+}$.

There are other definitions of the Hankel transform for distributions ([21], p. 191). We use the definition which appears in [30], formula ((AI, 3, 3), p. 70).

The Laplace transform of radial functions

We define the n-dimensional Laplace integral:

$$L[F] = f(z_1, \dots, z_n) = \int_{R^n} e^{-i<x,z>} F(r^2) dx,$$

here

$$< x, z >= x_1 z_1 + \cdots + x_n z_n.$$

The complex version of the Bochner formula is due to Alberto González Domínguez (cf. [26]). He extends the classical Bochner theorem to the complex case.

This result permits evaluate the n-dimensional Laplace integrals of radial functions by means of a simple integral.

Theorem. (A. G. Domínguez)

Hypothesis:
$F(r^2) \in D_{R^n}$.

Thesis: The following formula is valid

$$L[F] = \varphi(z_1^2 + \cdots + z_n^2) = \frac{(2\pi)^{\frac{n}{2}}}{(z_1^2 + \cdots + z_n^2)^{\frac{n-2}{2}}} \int_0^\infty F(t^2) t^{\frac{n}{2}} J_{\frac{n-2}{2}}[t\sqrt{z_1^2 + \cdots + z_n^2}] dt.$$

We note that if $\tau_j = 0, j = 0, 1, \dots, n$, the thesis of the Theorem becomes the classical Bochner Theorem (cf. [16], p. 187).

Now we shall establish the two following representation formulae.

Theorem. (S. E. Trione)

Hypothesis:
$F(r^2) \in D_{R^n}$.

Thesis:

a) If $n = 2m + 2, \quad m = 0, 1, \dots,$

$$L[F] = (-1)^m 2^{2m+1} \pi^{m+1} \frac{d^m}{ds^m} \int_0^\infty F(t^2) t J_0(\sqrt{st}) dt.$$

b) If $n = 2m + 1, \quad m = 1, 2, \dots,$

$$L[F] = (-1)^m 2^{2m+1} \pi^m \int_0^\infty F(t^2) t \cos(t\rho) dt.$$

The thesis a) and b) of the above Theorem are analogous of the radial formulas due to Leray ([15], p. 41, formulae (19,11)).

Here $*$ designates, as usual, the convolution.

Schwartz ([4], p. 264) has evaluated the Fourier transforms, of the function $R_\alpha(x, n)$ of Riesz, by evaluating their Laplace transform (first step), and then passing to the limit (in S') for $y \to 0$, where $y \in \nu$, (second step). The method was later employed by Lavoine [31] and Vladimirov [32]. It works generally for any $\phi(t) \in \mathcal{R}$ which is, besides, a continuous function of slow growth. It is clear that the basic formula greatly facilitates the use of Schwartz' method, since it disposes of its first step.

Other version.

We evaluate the Fourier transforms of retarded Lorentz-invariant functions (and distributions) as limits of Laplace transforms. Our method works generally for any retarded Lorentz-invariant functions $\phi(t)(t \in R^n)$ which is, besides, a continuous function of slow growth.

Appendix: Notations and Definitions

I.1 Let x be a point of the Euclidean space R^1 and λ be a complex number.

Consider the following functions $x_+^\lambda, x_-^\lambda, |x|^\lambda sgnx$ (cf. [1]), defined as follows:

$$x_+^\lambda = \begin{cases} x^\lambda & \text{for } x > 0, \\ 0 & \text{for } x \leq 0. \end{cases}$$

$$x_-^\lambda = \begin{cases} |x|^\lambda & \text{for } x < 0, \\ 0 & \text{for } x \geq 0. \end{cases}$$

$$|x|^\lambda = x_+^\lambda + x_-^\lambda,$$

$$|x|^\lambda sgnx = x_+^\lambda - x_-^\lambda,$$

here is $Re\lambda \geq 1$.

The function $\Gamma(\alpha)$ is defined by the formula

$$\Gamma(\alpha) = \int_0^\infty e^{-x} x^{\alpha-1} dx,$$

$Re\ \alpha > 0$.

We shall define the following generalized functions,

$$\frac{x_+^{\alpha-1}}{\Gamma(\alpha)}, \frac{x_-^{\alpha-1}}{\Gamma(\alpha)}, \frac{|x|^{\alpha-1}}{\Gamma(\frac{\alpha}{2})} \text{ and } \frac{|x|^{\alpha-1}\ sgn\ x}{\Gamma(\frac{\alpha+1}{2})}.$$

The above distributions are the regularized functions of $x_+^\lambda, x_-^\lambda, |x|^\lambda$ and $|x|^\lambda\ sgn\ x$.

Now, we define two generalized functions $(x+i0)^\lambda$ and $(x-i0)^\lambda$ as follows ([1], p. 59):

$$(x+i0)^\lambda = \lim_{y\to 0}(x+iy)^\lambda = \begin{cases} x^\lambda & \text{if } x > 0, \\ |x|^\lambda e^{i\lambda\pi} & \text{if } x < 0. \end{cases}$$

Similarly,

$$(x-i0)^\lambda = \lim_{y\to 0}(x-iy)^\lambda = \begin{cases} x^\lambda & \text{if } x > 0, \\ |x|^\lambda e^{-i\lambda\pi} & \text{if } x < 0. \end{cases}$$

These functions are defined for all complex λ.

We, obviously, may write

$$(x+i0)^\lambda = x_+^\lambda + e^{i\lambda\pi}x_-^\lambda,$$

and

$$(x-i0)^\lambda = x_+^\lambda + e^{-i\lambda\pi}x_-^\lambda.$$

We also have

$$(x+i0)^{-n} = x^{-n} - \frac{i\pi(-1)^{n-1}}{(n-1)!}\delta^{(n-1)}(x),$$

and

$$(x-i0)^{-n} = x^{-n} + \frac{i\pi(-1)^{n-1}}{(n-1)!}\delta^{(n-1)}(x).$$

1) $P_v^\mu(z) = \frac{1}{\Gamma(1-\mu)}(\frac{z+1}{z-1})^{\frac{\mu}{2}} \; {}_2F_1\,(-v, v+1; 1-\mu; \frac{1}{2}-\frac{1}{2}z)$ (Legendre function) where ${}_2F_1\,(a,b;c;z)$ (Gauss' hypergeometric series).

2) ${}_mF_n(a_1,\ldots,a_n;\gamma_1,\ldots,\gamma_n;z) = \sum_{k=0}^{\infty}\frac{(a_1)_k\ldots(a_m)_k z^k}{(\gamma_1)_k\ldots(\gamma_m)_k k!}$, $(a)_\nu = \frac{\Gamma(a+\nu)}{\Gamma(a)}$, $(a)_0 = 1$, $(a)_n = a(a+1)\ldots(a+n-1)$.

3) $L_k^\alpha(z) = \frac{e^z z^{-\alpha}}{k!}\frac{d^k}{dz^k}(e^{-z}z^{\alpha+k})$ (Laguerre polynomial); $L_k^0(z) = L_k(z)$.

4) $M_{k,\mu}(z) = z^{\mu+\frac{1}{2}}e^{-\frac{z}{2}}\;{}_1F_1(\frac{1}{2}+\mu-k; z_\mu+1; z)$ (Whittaker's functions); ${}_1F_1(a;c;z)$ (Kummer's confluent hypergeometric series).

5) $\Psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$

I.2 Let $x = (x_1, x_2, \dots x_n)$ and $y = (y_1, y_2, \dots y_n)$ be two points of the n-dimensional Euclidean space R^n.

We shall write

$$dx = dx_1, dx_2, \dots, dx_n, \tag{A,1}$$

$$< x, y >= \sum_{i=1}^{n} x_i y_i, \tag{A,2}$$

$$|x|^2 = r = x_1^2 + \cdots + x_n^2, \tag{A,3}$$

$$|y|^2 = \rho = y_1^2 + \cdots + y_n^2. \tag{A,4}$$

The Fourier transform of $f(x)$ is, by definition,

$$\mathcal{F}[f(x)] = g(y) = \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{R^n} e^{-i<x,y>} f(x) dx. \tag{A,5}$$

Let $z = (z_1, z_2, \dots z_n)$ be a point of the n-dimensional complex space C^n, where $z_\nu = x_\nu + iy_\nu, \nu = 1, 2, \dots, n$,

$$< x, z >= \sum_{i=1}^{n} x_i z_i. \tag{A,6}$$

The Laplace transform of $f(x)$ is, by definition,

$$L[f] = \int_{R^n} e^{-i<x,z>} f(x) dx. \tag{A,7}$$

Consider a non-degenerate quadratic form in n variables of the form

$$P = P(x) = x_1^2 + \cdots + x_p^2 - x_{p+1}^2 - x_{p+1}^2 - \cdots - x_{p+q}^2, \tag{A,8}$$

where $n = p + q$. The distribution $(P \pm i0)^\lambda$ is defined by

$$(P \pm i0)^\lambda = \lim_{\varepsilon \to 0} (P \pm i\varepsilon |x|^2)^\lambda, \tag{A,9}$$

where $\varepsilon > 0, |x|^2 = x_1^2 + \cdots + x_n^2, \quad \lambda \in C.$

The distributions $(P \pm i0)^\lambda$ are an important contribution of Gelfand ([ ], p.274).

These generalized functions can be expressed as

$$(P \pm i0)^\lambda = P_+^\lambda + e^{\pm i\pi\lambda} P_-^\lambda, \tag{A,10}$$

where

$$P_+^\lambda = \begin{cases} P^\lambda & \text{if } P \geq 0, \\ 0 & \text{if } P < 0; \end{cases} \qquad (A,11)$$

$$P_-^\lambda = \begin{cases} 0 & \text{if } P \geq 0, \\ (-P)^\lambda & \text{if } P < 0. \end{cases} \qquad (A,12)$$

Similarly, we have

$$(Q \pm i0)^\lambda = \lim_{\varepsilon \to 0} (Q \pm i\varepsilon |y|^2)^\lambda, \qquad (A,13)$$

where $\varepsilon > 0$, $|y|^2 = y_1^2 + \cdots + y_n^2$, $\lambda \in C$.

Here

$$Q = Q(y) = y_1^2 + \cdots + y_p^2 - y_{p+1}^2 - \cdots - y_{p+q}^2, \qquad (A,14)$$

$p + q = n$

The distributions $(m^2 + P \pm i0)^\lambda$ are defined in an analogue manner as the distributions $(P \pm i0)^\lambda$ (cf [1], p. 289).

$$(m^2 + Q \pm i0)^\lambda = \lim_{\varepsilon \to 0} (m^2 + Q \pm i\varepsilon |x|^2)^\lambda. \qquad (A,15)$$

It is useful to state an equivalent definition of the distributions $(m^2 + Q \pm i0)^\lambda$.

In this definition appear the distributions

$$(m^2 + Q)_+^\lambda = \begin{cases} (m^2 + Q) & \text{if } m^2 + Q \geq 0, \\ 0 & \text{if } m^2 + Q < 0. \end{cases} \qquad (A,16)$$

$$(m^2 + Q)_-^\lambda = \begin{cases} 0 & \text{if } m^2 + Q > 0, \\ (-m^2 - Q)^\lambda & \text{if } m^2 + Q \leq 0. \end{cases} \qquad (A,17)$$

We can prove, without difficulty, that the following formula is valid ([9], p. 566)

$$(m^2 + Q \pm i0)^\lambda = (m^2 + Q)_+^\lambda + e^{\pm i\pi\lambda} (m^2 + Q)_-^\lambda. \qquad (A,18)$$

From this formula we conclude immediately that

$$(m^2 + Q + i0)^\lambda = (m^2 + Q - i0)^\lambda = (m^2 + Q)^\lambda, \qquad (A,19)$$

when $\lambda = k =$ positive integer.

We observe that $(m^2 + Q \pm i0)^\lambda$ are entire distributional functions of λ. This is the principal difference between the distributions, formally analogue $(Q \pm i0)^\lambda$ which have poles at the points $\lambda = -\frac{n}{2} - k$, $k = 0, 1, \ldots$.

It can be proved (cf. [35], p. 573, formula (2.14) and p. 579, formula (3.5)) that

$$(m^2+Q\pm i0)^{-k} = Pf(m^2+Q)^{-k} \mp \frac{i\pi(-1)^{k-1}}{(k-1)!}\delta^{(k-1)}(m^2+Q), \qquad (A,20)$$

$k=0,1,\ldots$.

Let $f(z,\lambda), z\in C$, be an entire functions of the variables z,λ:

$$f(z,\lambda)=\sum_{\nu=0}^{\infty} a_\nu(\lambda) z^\nu. \qquad (A,21)$$

Let us consider the family of distributions of the form ([1], p. 285)

$$T(P\pm i0,\lambda)=(P\pm i0)^\lambda f(P\pm i0,\lambda)=(P\pm i0)^\lambda \sum_{\nu=0}^{\infty} a_\nu(\lambda)(P\pm i0)^\nu. \qquad (A,22)$$

Further, we consider the rotation-invariant distributions

$$T(|x|^2,\lambda)=(|x|^2)\sum_{\nu=0}^{\infty} a_\nu(\lambda)|x|^{2\nu}. \qquad (A,23)$$

We have (cf. [3], p. 25, formula (I,4,11)) the following

Theorem: Let $T(P\pm i0,\lambda)$ be a temperate distribution, then the following formula is valid

$$\mathcal{F}[T(P\pm i0)^\lambda] = e^{\mp i\frac{\pi}{2}q}[T(|x|^2,\lambda)]^\wedge\Big|_{|y|^2\to Q\mp i0}. \qquad (A,24)$$

The symbol which appears on the right-hand side has the following meaning: first, we evaluate the Fourier transform of $T(|x|^2,\lambda)$ and then we replace $|y|^2$ by $(Q\mp i0)$.

We observe that in the preceding theorem, $T(|x|^2,\lambda)$ is a temperate rotation-invariant distribution, therefore its Fourier transform can be evaluated by the distributional Bochner formula ([30], Theorem 26, p. 72).

$$\mathcal{F}\{T(|x|^2,\lambda)\}^\wedge = \{H\{\tilde{T}(|\tilde{x}|^2,\lambda)\}\}.$$

	$f(x)$	$\hat{f}(\xi) = \int_R e^{-2\pi i\xi x} f(x)dx$
(1)	$H(x)$	$\frac{1}{2\pi i} vp\frac{1}{\xi} + \frac{1}{2}\delta(\xi)$
(2)	$H(x)e^{-\lambda x}$ $(\lambda \in R)$	$\frac{1}{\lambda + 2\pi i\xi}$
(3)	$vp\frac{1}{x}$	$-i\pi\ sgn\ \xi$
(4)	$\|x\|$	$-\frac{1}{2\pi^2} pf\frac{1}{\xi^2}$
(5)	$X_{[-T,T]}.sgn\ x$ [X : characteristic function]	$\frac{1-\cos 2\pi T\xi}{i\pi\xi}$
(6)	$X_{[-T,T]}\, ln\,\|x\|$ $[T > 0]$	$ln\,T.\frac{sen 2\pi T\xi}{\pi\xi} - \frac{S_i(2\pi T\xi)}{\pi\xi}$, where $S_i(\xi) = \int_0^\xi \frac{sen t}{t} dt$.
(7)	$th\ x$	$\frac{-i\pi\xi}{sh\ \pi^2\xi}\ vp\ \frac{1}{\xi}$
(8)	$S_i(2\pi Tx)$	$-\frac{i}{2} X_{[-T,T]}\ vp\ \frac{1}{\xi}$
(9)	$\frac{1-\cos x}{sh x}$	$-\frac{i}{\pi^3}\{th\frac{\xi}{\pi^2} - \frac{1}{2}[th\frac{1}{\pi^2}(\xi - \frac{1}{2\pi}) + th\frac{1}{\pi^2}(\xi + \frac{1}{2\pi})]\}$

Fourier Transforms in R

(10)	$\coth x$	$-i\coth \pi\xi$
(11)	$\frac{1}{ch\, ax}\ vp\ \frac{1}{x}$ $[a > 0]$	$4i\{\frac{\pi}{4} - arctg\ e^{2\pi x}\}$
(12)	$\frac{1}{ch^2 x}$	$\frac{2\pi^2\xi}{sh\ \pi^2\xi}$
(13)	$pf\ \frac{1}{sh\ x}$	$-\frac{i}{\pi^3}\ th\ \frac{\xi}{\pi^2}$
(14)	$pf\ \frac{1}{\lvert x\rvert}$	$C - 2\ln\lvert x\rvert$ where $C = -2\gamma - 2\ln 2\pi$; γ: Euler's constant.
(15)	$arctg\ x$	$-\frac{i}{2}\ pf\ \frac{e^{-2\pi\lvert\xi\rvert}}{\xi}$

Fourier Transforms in R

(16)	$f(x)$	$\int_R e^{-i\xi x} f(x)dx$
(17)	$x^m e^{ibx}$	$2\pi i^m \delta^{(m)}(\xi - b)$
(18)	$vp\ \frac{1}{x^{m+1}}$	$\frac{(-i)^{m+1}\pi}{m!}\ \xi^m\ sgn\ \xi$
(19)	$x^\beta H(x) \quad [\beta \neq Z]$	$\beta!\lvert\xi\rvert^{-\beta-1} e^{-(\pi/2)i(\beta+1)\ sgn\ \xi}$
(20)	$\lvert x\rvert^\beta \quad [\beta \neq Z]$	$\beta!\lvert\xi\rvert^{-\beta-1} 2. \cos\frac{\pi}{2}(\beta+1)$
(21)	$ln\lvert x\rvert$	$-\pi\ vp\ \frac{1}{\xi}$
(22)	$vp\ \frac{1}{x^{m+1}}.\ sgn\ x$	$\frac{(-i\alpha)^m}{m!}(C - 2\ln\lvert\xi\rvert)$
(23)	$\lvert x\rvert^\beta\ sgn\ x \quad [\beta \neq Z]$	$\beta!\lvert\xi\rvert^{-\beta-1}(-2i) sen\frac{\pi}{2}(\beta+1) sgn\ \xi$
(24)	$x^\beta H(x)\ ln\, x \quad [\beta \neq Z]$	$\beta!\lvert\xi\rvert^{-\beta-1}\{\psi(\beta) - ln\lvert\xi\rvert - \frac{\pi}{2} i\ sgn\ \xi\}.e^{-\frac{\pi}{2}i(\beta+1) sgn\ \xi}$
(25)	$\lvert x\rvert^\beta\ ln\lvert x\rvert \quad [\beta \neq Z]$	$\beta!\lvert\xi\rvert^{-\beta-1} 2. \cos\frac{\pi}{2}(\beta+1)\{\psi(\beta) - ln\lvert\xi\rvert - \frac{\pi}{2} tg\frac{\pi}{2}(\beta+1)\}$
(26)	$\lvert x\rvert^\beta\ ln\lvert x\rvert\ sgn\ x \quad [\beta \neq Z]$	$\beta!\lvert\xi\rvert^{-\beta-1}(-2i) sen\frac{\pi}{2}(\beta+1)\{\psi(\beta) - ln\lvert\xi\rvert + \frac{\pi}{2} ctg\frac{\pi}{2}(\beta+1)\} sgn\ \xi$

$\psi(z) = \frac{\Gamma'(z)}{\Gamma(z)}$.

Fourier Transforms in R

(27)	$vp\ \frac{1}{x^{m+1}}.\ln\|x\|$	$\frac{(-1)^{m+1}\pi\xi^m}{m!}e^{[\frac{(m+1)}{2}]\pi i}.\{\psi(m)-\ln\|\xi\|\}sgn\ \xi$
(28)	$vp\ \frac{1}{x}.\ln\|x\|$	$(\ln\|\xi\|+\gamma)^2+C$
(29)	$vp\ \frac{1}{x^{m+1}}.\ln\|x\|sgn\ x$	$\frac{(-i\xi)^m}{m!}[\{\ln\|\xi\|-\psi(m)\}^2+C]$
(30)	$x^m\ln\|x\|sgn\ x$	$m!2(i\xi)^{-m-1}\{\psi(m)-\ln\|\xi\|\}$
(31)	$e^{-\alpha x}x^\beta H(x)\quad (Re\beta>-1)$	$\frac{\beta!}{(\alpha+i\xi)^{\beta+1}}$
(32)	$\frac{x_+^{\alpha-1}}{\Gamma(\alpha)}$	$e^{-i\frac{\pi}{2}\alpha}(y-i0)^{-\alpha}$
(33)	δ	1
(34)	$\delta^{(n)}$	$e^{i\frac{\pi}{2}n}y^n=(iy)^n$
(35)	$\frac{\cos(a\sqrt{y^2+b^2})}{(y^2+b^2)^{3/2}}\quad a,b>0\quad \nu>a$	$\frac{\pi}{b}e^{-\nu b}$

Fourier Transforms in R^n

	$f(x)$	$\mathcal{F}[f(x)] = \int_{R^n} f(x) e^{-<\xi,x>} dx$
(36)	$e^{-\frac{r^2}{2}}$	$(2\pi)^{\frac{n}{2}} e^{-\frac{\xi^2}{2}}$
(37)	$r^{\beta} \quad (\beta \neq 2m \text{ and } \beta \neq -n-2m)$	$\frac{(\frac{1}{2}\beta+\frac{1}{2}n-1)!}{(-\frac{1}{2}\beta-1)!} 2^{\beta+n} \pi^{\frac{n}{2}} \xi^{-\beta-n}$
(38)	r^{2m}	$(2\pi)^n (-1)^m (\partial_1^2 + \cdots + \partial_n^2)^m \delta(\xi)$
(39)	r^{-n-2m}	$\frac{(-1)^{m+1} \pi^{\frac{n}{2}} \xi^{2m}}{(\frac{n}{2}+m-1)! m! 2^{2m-1}} (ln\, \xi + C)$
(40)	$ln(\frac{1}{r})$	$(\frac{n}{2}-1)! 2^{n-1} \pi^{\frac{n}{2}} \xi^{-n}$
(41)	$r^{\beta}\, ln\, r \quad (\beta \neq 2m \text{ and } \beta \neq -n-2m)$	$\frac{(\frac{1}{2}\beta+\frac{n}{2}-1)!}{(-\frac{1}{2}\beta-1)!} 2^{\beta+n} \pi^{\frac{n}{2}} \xi^{-\beta-n} . \{\frac{1}{2}\psi(\frac{\beta}{2}+\frac{n}{2}-1) + \frac{1}{2}\psi(-\frac{\beta}{2}-1) - ln\, \frac{\xi}{2}\}$
(42)	$r^{2m}\, ln\, r$	$(\frac{n}{2}+m-1)! m! 2^{2m+n-1} \pi^{\frac{n}{2}} (-1)^{m+1} \xi^{-n-2m}$
(43)	$r^{-n-2m}\, ln\, r$	$\frac{(-1)^m \pi^{\frac{n}{2}} \xi^{2m}}{(\frac{n}{2}+m-1)! m! 2^{2m}} . \{ln\, \frac{\xi}{2} - \frac{1}{2}\psi(m) - \frac{1}{2}\psi(\frac{n}{2}+m-1)\}^2 + C$

(44)	$u_1^\beta \quad (\beta \neq \pm 2m)$	$\{(\frac{1}{2}\beta)!\}^2 2^{\beta+2}\{u_1^{-\beta-2} \cos \pi(\frac{\beta}{2}+1) + u_3^{-\beta-2}\}$
(45)	u_1^{2m}	$\frac{(-1)^m}{2^{2m}}(\partial_1^2 - \partial_2^2)^m\{\pi^2\delta(\xi) - 2U^{-2}\}$
(46)	u_1^{-2-2m}	$\frac{(-1)^{m+1}U^{2m}}{(m!)^2 2^{2m+2}}.\{\pi^2 \ sgn \ U^2 - (c_1 - 2\ln\|\xi_1+\xi_2\|).(c_2 - 2ln\|\xi_1-\xi_2\|)\}$
(47)	$u_2^\beta \quad [\beta \neq \pm 2m]$	$\{(\frac{\beta}{2})!\}^2 2^{\beta+1}\|U\|^{-\beta-2}e^{-\pi i[(\frac{\beta}{2})+1]H(U^2) sgn \ \xi_1}$
(48)	u_2^{2m}	$\frac{(-1)^m}{2^{2m+1}}(\partial_1^2 - \partial_2^2)^m\{\frac{1}{2}\pi^2\delta(\xi) - i(\xi_1+\xi_2^{-1}\delta(\xi_1-\xi_2) -$ $-i(\xi_1-\xi_2)^{-1}\delta(\xi_1+\xi_2) - U^{-2}\}.$
(49)	u_2^{-2-2m}	$\frac{(-1)^{m+1}U^{2m}}{(m!)^2 2^{2m+2}}\{\pi sgn \ (\xi_1+\xi_2) + i(C_1 - 2\ln\|\xi_1+\xi_2\|)\}.$ $.\{\pi sgn \ (\xi_1-\xi_2) + i(C_2 - 2\ln\|\xi_1-\xi_2\|)\}$
(50)	$u_3^\beta \quad (\beta \neq \pm 2m)$	$\{(\frac{\beta}{2})!\}^2 2^{\beta+2}\{U_1^{-\beta-2} + U_3^{-\beta-2} \cos \pi(\frac{\beta}{2}+1)\}$

$u_1^\beta = |u|^\beta H(u^2)$
$u_2^\beta = |u|^\beta H(u^2)H(x_1)$
$u_3^\beta = |u|^\beta H(-u^2)$
c_1, c_2, C_1 and C_2 arbitrary constants

(51)	u_3^{-2-2m}	$\frac{U^{2m}}{(m!)^2 2^{2m+2}}\{\pi^2 sgn\ U^2 + (c_1 - 2\ln\lvert\xi_1+\xi_2\rvert).(c_2 - 2\ln\lvert\xi_1-\xi_2\rvert)\}$
(52)	$u_4^\beta \quad [\beta \neq \pm 2m]$	$\{(\frac{\beta}{2})!\}^2 2^{\beta+1}\lvert U\rvert^{-\beta-2} e^{\pi i[(\frac{\beta}{2})+1]H(u^2) sgn \xi_1}$
(53)	u_4^{-2-2m}	$\frac{(-1)^{m+1}U^{2m}}{(m!)^2 2^{2m+2}}\{\pi\ sgn(\xi_1+\xi_2) - i(c_1 - 2\ln\lvert\xi_1+\xi_2\rvert)\}.$ $.\{\pi\ sgn(\xi_1-\xi_2) - i(c_2 - 2\ln\lvert\xi_1-\xi_2\rvert)\}$
(54)	$\lvert u\rvert^\beta \quad [\beta \neq \pm 2m]$	$\{(\frac{\beta}{2})!\}^2 2^{\beta+2}\lvert U\rvert^{-\beta-2}\{1+\cos\pi(\frac{\beta}{2}+1)\}$
(55)	$\lvert u\rvert^{-2-2m}$	$\frac{(-1)^{m+1}\pi^2\lvert U\rvert^{2m}}{(m!)^2 2^{2m+1}}$ $(m\ odd)$ $\frac{(-1)^m U^{2m}}{(m!)^2 2^{2m+1}}(c_1 - 2\ln\lvert\xi_1-\xi_2\rvert).(c_2 - 2\ln\lvert\xi_1+\xi_2\rvert)$ $(m\ even)$
(56)	$\lvert u\rvert^\beta sgn\ u^2 \quad [\beta \neq \pm 2m]$	$\{(\frac{\beta}{2})!\}^2 2^{\beta+2}\lvert U\rvert^{-\beta-2} sgn\ U^2\{\cos\pi(\frac{\beta}{2}+1) - 1\}$
(57)	$\lvert u\rvert^{-2-2m} sgn\ u^2$	$\frac{(-1)^m U^{2m}}{(m!)^2 2^{2m+1}}(c_1 - 2\ln\lvert\xi_1-\xi_2\rvert).(c_2 - 2\ln\lvert\xi_1+\xi_2\rvert)$ $(m\ odd)$ $\frac{(-1)^{m+1}\pi^2\lvert U\rvert^{2m} sgn U^2}{(m!)^2 2^{2m+1}}$ $(m\ even)$

$u_4^\beta = \lvert u\rvert^\beta H(u^2)H(-x_1)$
$\lvert u\rvert = \lvert u^2\rvert^{1/2}$

(58)	u_2^β $[\beta \neq \pm 2m; \beta \neq -n \pm 2m]$	$(\frac{1}{2}\beta)!(\frac{\beta}{2}+\frac{n}{2}-1)!2^{\beta+n-1}\pi^{\frac{n}{2}-1}\|U\|^{-\beta-n}.$ $.e^{-(\pi/2)i(\beta+n)H(U^2)sgn\ \xi_1}$
(59)	u_1^β $[\beta \neq \pm 2m; \beta+n \neq \pm 2m]$	$(\frac{\beta}{2})!(\frac{\beta}{2}+\frac{n}{2}+1)!2^{\beta+n}\pi^{\frac{n}{2}-1}\{U_1^{-\beta-n} \cos\frac{\pi}{2}(\beta+n)+U_3^{-\beta-n}\}$
(60)	u_3^β $[\beta \neq \pm 2m; \beta+n \neq \pm 2m]$	$(\frac{\beta}{2})!(\frac{\beta}{2}+\frac{n}{2}-1)!2^{\beta+n}\pi^{\frac{n}{2}-1}.$ $.\{U_1^{-\beta-n} \cos\frac{\pi}{2}(n+2)+U_3^{-\beta-n} \cos\frac{\pi}{2}(\beta+2)\}$
(61)	$u_1^\beta \ln\|u\|$ $[\beta \neq \pm 2m; \beta+n \neq \pm 2m]$	$(\frac{\beta}{2})!(\frac{\beta}{2}+\frac{n}{2}-1)!2^{\beta+n}\pi^{\frac{n}{2}-1}.[\frac{1}{2}\{\psi(\frac{\beta}{2})+\psi(\frac{\beta}{2}+\frac{n}{2}-1)+2\ln 2\}.$ $.\{U_1^{-\beta-n} \cos\frac{\pi}{2}(\beta+n)+U_3^{-\beta-n}\}-\frac{\pi}{2}U_1^{-\beta\frac{n}{2}} sen\frac{\pi}{2}(\beta+n)-$ $U_1^{-\beta-n} \cos\frac{\pi}{2}(\beta+n)\ln\|U\|-U_3^{-\beta-n}\ln\|U\|]$
(62)	$u_2^\beta \ln\|u\|$ $[\beta \neq \pm 2m; \beta+n \neq \pm 2m]$	$(\frac{\beta}{2})!(\frac{\beta}{2}+\frac{n}{2}-1)!2^{\beta+n-1}\pi^{\frac{n}{2}-1}.[\frac{1}{2}\{\psi(\frac{\beta}{2})+\psi(\frac{\beta}{2}+\frac{n}{2}-1)+2\ln 2\}.$ $.\|U\|^{-\beta-n}e^{-\frac{\pi}{2}i(\beta+n)H(u^2)sgn\ \xi_1}-\|U\|^{-\beta-n}\ln\|U\|.$ $.e^{-(\frac{\pi}{2})i(\beta+n)H(U^2)sgn\ xi_1}-\frac{1}{2}\pi i U_2^{-\beta-n}e^{-\frac{\pi}{2}i(\beta+n)}+\frac{\pi}{2}iU_4^{-\beta-n}e^{\frac{\pi}{2}i(\beta+n)}]$

$u^2 = x_1^2 - x_2^2 - \ldots - x_n^2, \quad (n > 2).$

(63) $u_3^\beta \, ln\,\lvert u\rvert \quad (\beta \neq \pm 2m; \beta + n \neq \pm 2m)$	$(\frac{\beta}{2})!(\frac{\beta}{2}+\frac{n}{2}-1)!2^{\beta+n}\pi^{\frac{n}{2}-1}.[\frac{1}{2}\{\psi(\frac{\beta}{2})+\psi(\frac{\beta}{2}+\frac{n}{2}-1)+2\,ln\,2\}.$ $.\{U_1^{-\beta-n}\cos\frac{\pi}{2}(n+2)+U_3^{-\beta-n}\cos\frac{\pi}{2}(\beta+2)\}-$ $-\frac{\pi}{2}U_3^{-\beta-n}\sin\frac{\pi}{2}(\beta+2)-U_1^{-\beta-n}\cos\frac{\pi}{2}(n+2)\,ln\,\lvert U\rvert-$ $-U_3^{-\beta-n}\cos\frac{\pi}{2}(\beta+2)\,ln\,\lvert U\rvert]$
(64) $H(u^2)$	$(\frac{n}{2}-1)!2^n\pi^{\frac{n}{2}-1}U_3^{-n},$ (n odd) $(\frac{n}{2}-1)!2^n\pi^{\frac{n}{2}-1}(-1)^{\frac{n}{2}}U^{-n}+2^{n-1}\pi^n\delta(\xi);$ (n even)
(65) $H(u^2)H(x_1)$	$(\frac{n}{2}-1)!2^{n-1}\pi^{\frac{n}{2}-1}(iU_1)^{-n},$ (n odd) $(\frac{n}{2}-1)!2^{n-1}\pi^{\frac{n}{2}-1}(-1)^{\frac{n}{2}}U^{-n}+(-1)^{\frac{n}{2}-1}2^{n-2}$ $\pi^{\frac{n}{2}}i\,sgn\,\xi_1\delta^{((\frac{n}{2})-1)}(U^2);$ (n even)
(66) $H(-u^2)$	$(\frac{n}{2}-1)!(-1)2^n\pi^{\frac{n}{2}-1}U_3^{-n},$ (n odd) $2^{n-1}\pi^n\delta(\xi)+(\frac{n}{2}-1)!2^n\pi^{\frac{n}{2}-1}.(-1)^{\frac{n}{2}-1}U^{-n};$ (n even)
(67) u_1^{2m}	$m!(\frac{n}{2}+m-1)!2^{n+2m}\pi^{\frac{n}{2}-1}U_3^{-n-2m},$ (n odd) $m!(\frac{n}{2}+m-1)!2^{n+2m}\{\pi^{\frac{n}{2}-1}(-1)^{n+m}.$ $.U^{-n-2m}+\frac{\frac{1}{2}\pi^n(-1)^m(\partial_1^2-\ldots\partial_n^2)^m\delta(\xi)}{(\frac{n}{2}-1)!}\};$ (n even)

(68)	u_2^{2m}	$m!(\frac{n}{2}+m-1)!2^{n+2m-1}\pi^{\frac{n}{2}-1}(iU_1)^{-n-2m}$; (n odd) $m!(\frac{n}{2}+m-1)!2^{n+2m-1}\pi^{\frac{n}{2}}(-1)^{\frac{n}{2}+m}.\{\frac{U^{-n-2m}}{\pi}-\frac{1}{2}i(\partial_1^2-\ldots-\partial_n^2)^m$ $sgn\ \xi_1.\frac{\delta^{[(\frac{n}{2})-1]}(U^2)}{(\frac{n}{2}-1)!}\}$; (n even)
(69)	u_3^{2m}	$m!(\frac{n}{2}+m-1)!2^{n+2m}\pi^{\frac{n}{2}-1}(-1)^{m+1}U_3^{-n-2m}$, (n odd) $m!(\frac{n}{2}+m-1)!2^{n+2m}\{\pi^{\frac{n}{2}-1}(-1)^{\frac{n}{2}-1}U^{-n-2m}+$ $+\frac{\frac{1}{2}\pi^n(\partial_1^2-\ldots-\partial_n^2)^m\delta(\xi)}{(\frac{n}{2}-1)}\}$; (n even)
(70)	u^{-2}	$(\frac{n}{2}-2)!2^{n-2}\pi^{\frac{n}{2}}(-1)^{\frac{n}{2}-\frac{1}{2}}U_1^{2-n}$ (n odd)
(71)	u_2^{2-n}	$\frac{2\pi^{\frac{n}{2}}(-1)^{\frac{n}{2}-\frac{1}{2}}U^{-2}}{(\frac{n}{2}-2)!}$ (n odd)
(72)	$\delta^{(m)}(u^2)$ $(0\leq m<\frac{n}{2}-1)$	$(\frac{n}{2}-m-2)!2^{n-2m-2}\pi^{\frac{n}{2}-1}U_3^{2n+2-n}$ (n odd) $(\frac{n}{2}-m-2)!2^{n-2m-2}(-1)^{\frac{n}{2}+m+1}\pi^{\frac{n}{2}-1}U^{2n+2-n}$ (n even)
(73)	$H(x_1)\delta^{(m)}(u^2)$ $(0\leq m\leq\frac{n}{2}-1)$	$(\frac{n}{2}-m-2)!2^{n-2m-3}\pi^{\frac{n}{2}-1}(-1)^{m+1}i^{-n}.U_1^{2m+2-n}$ (n odd) $(\frac{n}{2}-m-2)!(-1)^{\frac{n}{2}+m+1}2^{n-2m-3}\pi^{\frac{n}{2}-1}.U^{2m+2-n}+$ $+(-1)^{\frac{n}{2}-1}2^{n-2m-4}\pi^{\frac{n}{2}}.i\ sgn\ \xi_1\delta^{[(\frac{n}{2})-m-2]}(U^2)$ (n even)
(74)	$(u^2\pm i0)^{\frac{\beta}{2}}$ $(\beta\neq\pm 2m,\beta+n\neq\pm 2m)$	$\frac{(\frac{\beta}{2}+\frac{n}{2}-1)!}{(-\frac{\beta}{2}-1)!}2^{\beta+n}\pi^{\frac{n}{2}}e^{\mp\frac{\pi}{2}i(n-1)}.(U^2\mp i0)^{-\frac{\beta}{2}-\frac{n}{2}}$

Fourier Transforms in R^n

(75) u_1^{β} $(\beta \neq \pm 2m, \beta + n \neq \pm 2m)$	$(\frac{\beta}{2})!(\frac{\beta}{2}+\frac{n}{2}-1)!2^{\beta+n}\pi^{\frac{n}{2}-1}.$ $.\{U_1^{-\beta-n}\, cos\, \frac{\pi}{2}(n-p+1+\beta)+U_3^{-\beta-n}\, cos\, \frac{\pi}{2}(\beta-1)\}$
(76) u_3^{β} $(\beta \neq \pm 2m, \beta + n \neq \pm 2m)$	$(\frac{\beta}{2})!(\frac{\beta}{2}+\frac{n}{2}-1)!2^{\beta+n}\pi^{\frac{n}{2}-1}.$ $.\{U_1^{-\beta-n}\, cos\, \frac{\pi}{2}(n-p-1)+U_3^{-\beta-n}\, cos\, \frac{\pi}{2}(p+\beta)\}$
(77) $H(u^2)$	$(\frac{n}{2}-1)!2^n\pi^{\frac{n}{2}-1}(-1)^{\frac{p}{2}-\frac{1}{2}}U_3^{-n}$ $(n$ odd, p odd$)$ $(\frac{n}{2}-1)!2^n\pi^{\frac{n}{2}-1}(-1)^{\frac{(n-p+1)}{2}}U_1^{-n}$ $(n$ odd, p even$)$ $(\frac{n}{2}-1)!2^n\pi^{\frac{n}{2}-1}(-1)^{\frac{(n-p+1)}{2}}U^{-n}+2^{n-1}\pi^n\delta(\xi)$ $(n$ even, p odd$)$ $2^n\pi^{\frac{n}{2}}(-1)^{\frac{p}{2}-\frac{1}{2}}\delta^{[(\frac{n}{2})-1]}(U^2)$ $(n$ even, p even$)$
(78) $\delta^{(m)}(u^2)$ $(0 \leq m < \frac{n}{2}-1)$	$(\frac{n}{2}-m-2)!2^{n-2m-2}\pi^{\frac{n}{2}-1}.$ $.\{U_3^{2m+2-n}\, sin\, \frac{p}{2}\pi+U_1^{2m+2-n}\, sin\, \frac{\pi}{2}(n-p)\}$
(79) $(u^2 \pm i0)^{\frac{\beta}{2}}$ $(\beta \neq \pm 2m, \beta + n \neq \pm 2m)$	$\frac{(\frac{\beta}{2}+\frac{n}{2}-1)!}{(-\frac{\beta}{2}-1)!}2^{\beta+n}\pi^{\frac{n}{2}}e^{\mp\frac{\pi}{2}i(\beta-p)}.(U^2 \mp i0)^{-\frac{\beta}{2}-\frac{n}{2}}$

$u^2 = x_1^2 + \cdots + x_p^2 - \cdots - x_n^2.$

	$f(x)$	$\mathcal{F}[f(x)] = \frac{1}{(2\pi)^{n/2}} \int_{R^n} e^{-i<x,y>} f(x) dx$	
(80)	$T(P \pm i0, \lambda)$	$e^{\pm i \frac{\pi}{2} q} \{T(\lvert x \rvert^2, \lambda)\}^{\wedge}\lvert_{\lvert y \rvert^2 \to Q \mp i0}$	(*)
(81)	$G_\alpha(P \pm i0, m, n)$	$\frac{1}{(2\pi)^{\frac{n}{2}}} e^{\frac{i\pi\alpha}{2}} (m^2 + Q \mp i0)^{-\frac{\alpha}{2}}$	
(82)	$G_{-2l}(P \pm i0, m, n)$	$\frac{(-1)^l (m^2+Q)^l}{(2\pi)^{n/2}}$	
(83)	$H_\alpha(P \pm i0, n)$	$\frac{e^{i\frac{\pi}{2}\alpha}}{(2\pi)^{\frac{n}{2}}} (Q \mp i0)^{-\frac{\alpha}{2}}$	
(84)	$H_{-2k}(P \pm i0, n)$	$\frac{(-1)^k}{(2\pi)^{\frac{n}{2}}} Q^k$	$k = 1, 2, \ldots$
(85)	$K^l\{\delta\}$	$\frac{1}{(2\pi)^{\frac{n}{2}}} (-1)^l (m^2 + Q)^l$	$l = 0, 1, \ldots$
(86)	$L^k\{\delta\}$	$\frac{1}{(2\pi)^{\frac{n}{2}}} (-1)^k Q^k$	$k = 0, 1, \ldots$
(87)	$Pf\ H_{n+2h}(P \pm i0, n)$	$\frac{(-1)^h e^{\pm i \frac{\pi}{2} q} (P \pm i0)^h}{\pi^{\frac{n}{2}} 2^{n+2h+1}} \cdot \left[\frac{2 \log 2 \Gamma(h+1)\Gamma(\frac{n+2h}{2}) + \Gamma'(h+1)\Gamma(\frac{n+2h}{2}) + \Gamma(h+1)\Gamma'(\frac{n+2h}{2})}{\{\Gamma(h+1)\Gamma(\frac{n+2h}{2})\}^2}\right] +$ $+ \frac{(-1)^{h+1} 2 e^{\pm i \frac{\pi}{2} q}}{\pi^{\frac{n}{2}} 2^{n+2h} \Gamma(h+1)\Gamma(\frac{n+2h}{2})} (P \pm i0)^h \log(P \pm i0)^{1/2}$	
(88)	$Pf R_n(x, n)$	$\frac{(-1)}{\pi^{n/2} 2^{n-1} \Gamma(\frac{n}{2})} \log r + \frac{1}{\pi^{n/2} 2^{n/2}} \left[\frac{2 \log 2 \Gamma(\frac{n}{2}) + \Gamma'(1)(\frac{n}{2}) + \Gamma'(\frac{n}{2})}{[\Gamma(\frac{n}{2})]^2}\right]$	
(89)	δ	$\frac{1}{(2\pi)^{\frac{n}{2}}}$	

(*) The symbol which appears on the right-hand side has the following meaning: first, we evaluate the Fourier transform of $T(|x|^2, \lambda)$ and then we replace $|y|^2$ by $(Q \mp i0)$.

$f(x)$	$\mathcal{F}[f(x)] = \int_{R^n} e^{-i<x,y>} f(x)dx$
(90) $G_R(t,\alpha,m^2,n) = \frac{(u-m^2)_+^{\alpha-1}}{\Gamma(\alpha)}$ (*)	$2^\alpha(2\pi)^{\frac{n-2}{2}} m^{\alpha+\frac{n-2}{2}} \theta(Q) \frac{K_{\alpha+\frac{n-2}{2}}(mQ^{1/2})}{(Q^{1/2})^{\alpha+(\frac{n-2}{2})}} +$ $+2^\alpha(2\pi)^{\frac{n-2}{2}} m^{\alpha+\frac{n-2}{2}} \theta(-Q)\theta(x_0).$ $\cdot\frac{K_{\alpha+\frac{n-2}{2}}(me^{i\frac{\pi}{2}}(-Q)^{1/2})}{e^{i\frac{\pi}{2}(\alpha+\frac{n-2}{2})}[(-Q)^{1/2}]^{\alpha+\frac{n-2}{2}}} + 2^\alpha(2\pi)^{\frac{n-2}{2}} m^{\alpha+\frac{n-2}{2}} \theta(-Q)\theta(-x_0).$ $\cdot\frac{K_{\alpha+\frac{n-2}{2}}(me^{-i\frac{\pi}{2}}(-Q)^{1/2}}{e^{-i\frac{\pi}{2}\alpha+\frac{n-2}{2}}[(-Q)^{1/2}]^{\alpha+\frac{n-2}{2}}} +$ $+\frac{2^{\alpha+\frac{n-4}{2}}(2\pi)^{\frac{n-2}{2}}\pi\Gamma(\alpha+\frac{n-2}{2})(-1)i\ sgn\ x_0}{(\alpha+\frac{n-2}{2}1)!}\delta(Q)^{\alpha+\frac{n-2}{2}-1},$ if $2(\alpha+\frac{n-2}{2})$ is even and $Q = x_1^2 + \cdots + x_{n-1}^2 - x_0^2$

(*) $$\frac{(u-m^2)_+^{\alpha-1}}{\Gamma(\alpha)} = \begin{cases} \frac{(u-m^2)^{\alpha-1}}{\Gamma(\alpha)} & \text{if } u-m^2>0 \text{ and } t_0>0 \\ 0 & \text{if } t \text{ belongs to the complementary set.} \end{cases}$$

		$2^{\alpha}(2\pi)^{\frac{n-2}{2}}m^{\alpha+\frac{n-2}{2}}\theta(Q)\frac{K_{\alpha+\frac{n-2}{2}}(mQ^{1/2})}{(Q^{1/2})^{\alpha+\frac{n-2}{2}}}+$ $+2^{\alpha}(2\pi)^{\frac{n-2}{2}}m^{\alpha+\frac{n-2}{2}}e^{-i\frac{\pi}{2}(\alpha+\frac{n-2}{2}}.(-\frac{1}{2})i\pi e^{-i\frac{\pi}{2}(\alpha+\frac{n-2}{2}}.$ $.\theta(-Q)\theta(x_0)\frac{H^{(2)}_{\alpha+\frac{n-2}{2}}[m(-Q)^{1/2}]}{[(-Q)^{1/2}]^{\alpha+\frac{n-2}{2}}}+$ $+2^{\alpha}(2\pi)^{\frac{n-2}{2}}m^{\alpha+\frac{n-2}{2}}e^{i\frac{\pi}{2}(\alpha+\frac{n-2}{2}}.(\frac{1}{2})i\pi e^{i\frac{\pi}{2}(\alpha+\frac{n-2}{2})}.$ $.\theta(-Q)\theta(-x_0)\frac{H^{(1)}_{\alpha+\frac{n-2}{2}}[m(-Q)^{1/2}]}{[(-Q)^{1/2}]^{\alpha+\frac{n-2}{2}}}+$ $+2^{2\alpha+(\frac{n-4}{2})}(2\pi)^{\frac{n-2}{2}}(-i\pi).[\delta_{C_b}(Q)^{\alpha+\frac{n-4}{2}}-\delta_{C_f}(Q)^{\alpha+\frac{n-4}{2}}]$, if $2(\alpha+\frac{n-2}{2})$ is even;
(91)	$G_R(t,\alpha,m^2,n)$	$2^{\alpha}(2\pi)^{\frac{n-2}{2}}m^{\alpha+\frac{n-2}{2}}\theta(Q)\frac{K_{\alpha+\frac{n-2}{2}}(mQ^{1/2})}{(Q^{1/2})^{\alpha+\frac{n-2}{2}}}+$ $+\theta(-Q)\theta(x_0)2^{\alpha}(2\pi)^{\frac{n-2}{2}}m^{\alpha+\frac{n-2}{2}}e^{-i\frac{\pi}{2}(\alpha+\frac{n-2}{2}}.(-\frac{1}{2})i\pi e^{-i\frac{\pi}{2}(\alpha+\frac{n-2}{2}}.$ $.\frac{H^{(2)}_{\alpha+\frac{n-2}{2}}[m(-Q)^{1/2}]}{[(-Q)^{1/2}]^{\alpha+\frac{n-2}{2}}}+$ $+\theta(-Q)\theta(-x_0)2^{\alpha}(2\pi)^{\frac{n-2}{2}}m^{\alpha+\frac{n-2}{2}}e^{i\frac{\pi}{2}(\alpha+\frac{n-2}{2})}.(\frac{1}{2})i\pi e^{i\frac{\pi}{2}(\alpha+\frac{n-2}{2}}.$ $.\frac{H^{(1)}_{\alpha+\frac{n-2}{2}}[m(-Q)^{1/2}]}{[(-Q)^{1/2}]^{\alpha+\frac{n-2}{2}}}+$ $+2^{\alpha+(\frac{n-4}{2})}(2\pi)^{\frac{n-2}{2}}\Gamma(\alpha+\frac{n-2}{2}).\{[\theta(Q)Q]^{-(\alpha+\frac{n-2}{2})}-i(-1)^{\alpha+\frac{n}{2}-\frac{3}{2}},$ $sgn\ x_0[\theta(-Q)(-Q)]^{-(\alpha+\frac{n-2}{2})}\},$ if $2(\alpha+\frac{n-2}{2})$ is odd.

	$2^{\alpha}(2\pi)^{\frac{n-2}{2}}m^{\alpha+\frac{n-2}{2}}\theta(Q)\frac{K_{\alpha+\frac{n-2}{2}}(mQ^{1/2})}{(Q^{1/2})^{\alpha+\frac{n-2}{2}}}+$ $+2^{\alpha}(2\pi)^{\frac{n-2}{2}}m^{\alpha+\frac{n-2}{2}}\theta(-Q)\theta(x_0).\frac{K_{\alpha+\frac{n-2}{2}}(me^{i\frac{\pi}{2}}(-Q)^{1/2})}{e^{i\frac{\pi}{2}(\alpha+\frac{n-2}{2})}[(-Q)^{1/2}]^{\alpha+\frac{n-2}{2}}}+$ $+2^{\alpha}(2\pi)^{\frac{n-2}{2}}m^{\alpha+\frac{n-2}{2}}\theta(-Q)\theta(-x_0).\frac{K_{\alpha+\frac{n-2}{2}}(me^{-i\frac{\pi}{2}}(-Q)^{1/2})}{e^{-i\frac{\pi}{2}(\alpha+\frac{n-2}{2})}[(-Q)^{1/2}]^{\alpha+\frac{n-2}{2}}}+$ $+2^{2\alpha+\frac{n-4}{2}}(2\pi)^{\frac{n-2}{2}}\Gamma(\alpha+\frac{n-2}{2}).$ $.[\theta(Q)Q]^{-(\alpha+\frac{n-2}{2})}-i(-1)^{\alpha+\frac{n}{2}-\frac{3}{2}}\ sgn\ x_0[\theta(-Q)(-Q)]^{-\alpha+\frac{n-2}{2}},$ if $2(\alpha+\frac{n-2}{2})$ is odd and $Q=x_1^2+\cdots+x_{n-1}^2-x_0^2$

Here

$$sgn\ x_0=\begin{cases}1 & \text{if } x_0>0,\\ -1 & \text{if } x_0<0.\end{cases}$$

$G_R(t,\alpha=0,m^2,n=4)=$ (92) $=\delta_{C_b}(u-m^2)\quad(*)$	$\theta(Q)2\pi m\frac{K_1[mQ^{\frac{1}{2}}]}{Q^{\frac{1}{2}}}+\theta(-Q)\pi^2 im[\theta(x_0)H_1^{(2)}\frac{[m(-Q)^{\frac{1}{2}}]}{(-Q)^{\frac{1}{2}}}-$ $-\theta\frac{(-x_0)H_1^{(1)}[m(-Q)^{1/2}]}{(-Q)^{1/2}]}+2\pi^2 i[\delta_{c_f}(Q)-\delta_{c_b}(Q)].$
$G_R(t,\alpha=1,m^2,n)=$ (93) $=\{\begin{matrix}1 & \text{if } u-m^2>0 \text{ and } t_0>0\\ 0 & \text{if } t \text{ belongs to the complementary set}\end{matrix}.$	$\theta(Q)2(2\pi)^{\frac{n-2}{2}}m^{n/2}\frac{K_{n/2}[m(Q)^{1/2}]}{(Q^{1/2})^{n/2}}-$ $-\theta(-Q)\theta(x_0)(2\pi)^{\frac{n-2}{2}}m^{n/2}i\pi e^{-i\frac{\pi}{2}n}\frac{H^{(2)}_{n/2}[m(-Q)^{1/2}]}{[(-Q)^{1/2}]^{n/2}}+$ $+\theta(-Q)\theta(-x_0)(2\pi)^{\frac{n-2}{2}}m^{\frac{n}{2}}i\pi e^{i\frac{\pi}{2}n}\frac{H^{(1)}_{\frac{n}{2}}[m(-Q)^{1/2}]}{[(-Q)^{1/2}]^{n/2}}-$ $-(2\pi)^{\frac{n-2}{2}}2^{\frac{n-2}{2}}i\pi[\delta_{c_f}(Q)^{\frac{n-2}{2}}-\delta_{c_b}(Q)^{\frac{n-2}{2}}],$ if n is even, $\theta(Q)2(2\pi)^{\frac{n-3}{2}}m^{n/2}\frac{K_{n/2}[m(Q)^{1/2}]}{(Q^{1/2})^{n/2}}-$ $-\theta(-Q)\theta(x_0)(2\pi)^{\frac{n-2}{2}}m^{n/2}i\pi e^{-i\frac{\pi}{2}n}\frac{H^{(2)}_{n/2}[m(-Q)^{1/2}]}{[(-Q)^{1/2}]^{n/2}}+$ $+\theta(-Q)\theta(-x_0)(2\pi)^{\frac{n-2}{2}}m^{\frac{n}{2}}i\pi e^{i\frac{\pi}{2}n}H^{(1)}_{\frac{n}{2}}[m(-Q)^{1/2}]+$ $+(2\pi)^{\frac{n-2}{2}}2^{n/2}\Gamma(\frac{n}{2})\{[\theta(Q)Q]^{-\frac{n}{2}}-$ $-i(-1)^{\frac{n-3}{2}}\ sgn\ x_0[\theta(-Q)(-Q)]^{-\frac{n}{2}}\},$ if n is odd.

(*) C_b is the interior of the backward cone:
$C_b=\{x\in R^n/x_0^2-x_1^2\ldots-x_{n-1}^2>0,x_0<0.$
C_f is the interior of the forward cone:
$C_f=\{x\in R^n/x_0^2-x_1^2-\ldots-x_{n-1}^2>0,x_0>0.$

	$\theta(Q)2^{-k}(2\pi)^{\frac{n-2}{2}}m^{-k+\frac{n-2}{2}}\frac{K_{-k+\frac{n-2}{2}}[m(Q)^{1/2}]}{(Q^{1/2})^{-k+\frac{n-2}{2}}}+$ $+\theta(-Q)\theta(x_0)2^{-k}(2\pi)^{\frac{n-2}{2}}m^{-k+\frac{n-2}{2}}e^{-i\pi(-k+\frac{n-2}{2})}.$ $.(-\frac{1}{2})^{i\pi}\frac{H^{(2)}_{-k+\frac{n-2}{2}}[m(-Q)^{1/2}]}{((-Q)^{1/2})^{-k+\frac{n-2}{2}}}+$ $+\theta(-Q)\theta(x_0)2^{-k}(2\pi)^{\frac{n-2}{2}}m^{-k+\frac{n-2}{2}}e^{i\pi(-k+\frac{n-2}{2})}.$ $.\frac{1}{2}^{i\pi}\frac{H^{(1)}_{-k+\frac{n-2}{2}}[m(-Q)^{1/2}]}{((-Q)^{1/2})^{-k+\frac{n-2}{2}}}+$ $+2^{-2k+\frac{n-4}{2}}(2\pi)^{\frac{n-2}{2}}(-i\pi)\ sgn\ x_0\delta(Q)^{-k+\frac{n-4}{2}}$, if $-2k+n-2$ is even,
(94) $G_R(t,\alpha=-k,m^2,n)=\delta_R^{(k)}(u-m^2)$	
	$\theta(Q)2^{-k}(2\pi)^{\frac{n-2}{2}}m^{-k+\frac{n-2}{2}}\frac{K_{-k+\frac{n-2}{2}}[m(Q)^{1/2}]}{(Q^{1/2})^{-k+\frac{n-2}{2}}}+$ $-\theta(-Q)\theta(x_0)2^{-k-1}(2\pi)^{\frac{n-2}{2}}m^{-k+\frac{n-2}{2}}e^{-i\pi(-k+\frac{n-2}{2})}.$ $.i\pi\frac{H^{(2)}_{-k+\frac{n-2}{2}}[m(-Q)^{1/2}]}{((-Q)^{1/2})^{-k+\frac{n-2}{2}}}+$ $+\theta(-Q)\theta(x_0)2^{-k-1}(2\pi)^{\frac{n-2}{2}}m^{-k+\frac{n-2}{2}}e^{i\pi(-k+\frac{n-2}{2})}.$ $.i\pi\frac{H^{(1)}_{-k+\frac{n-2}{2}}[m(-Q)^{1/2}]}{((-Q)^{1/2})^{-k+\frac{n-2}{2}}}+$ $+2^{-2k+\frac{n-4}{2}}(2\pi)^{\frac{n-2}{2}}\Gamma(-k+\frac{n-2}{2}).[\theta(Q)Q]^{-(-k+\frac{n-2}{2})}+$ $+e^{i\pi(-k+\frac{n-2}{2})}\ sgn\ x_0[\theta(-Q)(-Q)]^{-(-k+\frac{n-2}{2})}$, if $-2k+n-2$ is odd.

$G_R(t,m^2=0,\alpha,n)=\frac{u_+^{\alpha-1}}{\Gamma(\alpha)}=$ (95) $=\{\begin{smallmatrix}\frac{u^{\alpha-1}}{\Gamma(\alpha)} & \text{if } u>0 \text{ and } t_0>0\\ 0 & \text{if } t \text{ belongs to the compl. set}\end{smallmatrix}$.	$(2\pi)^{\frac{n-2}{2}}2^{2\alpha+\frac{n-4}{2}}\Gamma(\alpha+\frac{n-2}{2}).$ $.\{\frac{-i\pi}{(\alpha+\frac{n}{2}-2)!}\ sgn\ x_0\delta(Q)^{\alpha+\frac{n-4}{2}}+$ $+(-1)^{\alpha+\frac{n-2}{2}}Pf\frac{1}{(-Q)^{\alpha+\frac{n-2}{2}}}\},$ if $2\alpha+n-2$ is even, $(2\pi)^{\frac{n-2}{2}}2^{2\alpha+\frac{n-4}{2}}\Gamma(\alpha+\frac{n-2}{2}).$ $.\{[\theta(Q)Q]^{-(\alpha+\frac{n}{2}-\frac{3}{2})-\frac{1}{2}}-i(-1)^{\alpha+\frac{n}{2}-\frac{3}{2}}.$ $.\ sgn\ x_0[\theta(-Q)(-Q)]^{-(\alpha+\frac{n}{2}-\frac{3}{2})-\frac{1}{2}}\},$ if $2\alpha+n-2$ is odd.
$G_R(t,\alpha=1,m^2=0,n)=$ (96) $=\{\begin{smallmatrix}1 & \text{if } u>0, t_0>0\\ 0 & \text{if } t \text{ belongs to the complementary set}\end{smallmatrix}$.	$(2\pi)^{\frac{n-2}{2}}2^{n/2}\Gamma(\frac{n}{2}).\{\frac{-i\pi}{(\frac{n}{2}-1)!}\ sgn\ x_0\delta(Q)^{\frac{n}{2}-1}+$ $+(-1)^{n/2}Pf\frac{1}{(-p)^{n/2}}\},$ if n is even, $2^{n-1}\pi^{n/2}\Gamma(\frac{n}{2})$ $\{[\theta(Q)Q]^{-\frac{n}{2}}-i(-1)^{\frac{n}{2}-\frac{1}{2}}\ sgn\ x_0[\theta(-Q)(-Q)]^{-\frac{n}{2}}\},$ if n is odd.
$G_R(t,\alpha=-k,m=0,n)=\delta_R^{(k)}(u),$ (97) $k\neq\frac{n-2}{2}+h,\quad h=0,1,\ldots$	$(2\pi)^{\frac{n-2}{2}}2^{-2k+\frac{n-4}{2}}\Gamma(-k+\frac{n-2}{2}).$ $\{\frac{-i\pi}{(-k+\frac{n}{2}-2)!}\ sgn\ x_0\delta(Q)^{-k+\frac{n-2}{2}}+$ $+(-1)^{-k+\frac{n-2}{2}}Pf\frac{1}{(-Q)^{-k+\frac{n-2}{2}}}\},$ where n is even and $-k+\frac{n-2}{2}\neq -l, l=0,1,\ldots,$ $(2\pi)^{\frac{n-2}{2}}2^{-2k+\frac{n-4}{2}}\Gamma(-k+\frac{n-2}{2}).$ $\{[\theta(Q)Q]^{-(-k+\frac{n}{2}-\frac{3}{2})-\frac{1}{2}}-i(-1)^{-k+\frac{n}{2}-\frac{3}{2}}.$ $.\ sgn\ x_0[\theta(-Q)(-Q)]^{-(-k+\frac{n}{2}-\frac{3}{2})-\frac{1}{2}},$ where n is odd.

$G_A(t,\alpha,m^2,n)=\frac{(u-m^2)_-^{\alpha-1}}{\Gamma(\alpha)}=$ (98) $=\begin{cases}\frac{(u-m^2)^{\alpha-1}}{\Gamma(\alpha)} & \text{if } u-m^2>0 \text{ and } t_0<0,\\ 0 & \text{if } t \text{ belongs to the compl. set}\end{cases}.$	$\theta(Q)2^\alpha(2\pi)^{\frac{n-2}{2}}m^{\alpha+\frac{n-2}{2}}\frac{K_{\alpha+\frac{n-2}{2}}[mQ^{1/2}]}{[Q^{1/2}]^{\alpha+\frac{n-2}{2}}}+$ $+\theta(-Q)\theta(x_0)2^\alpha(2\pi)^{\frac{n-2}{2}}m^{\alpha+\frac{n-2}{2}}.$ $\cdot\frac{K_{\alpha+\frac{n-2}{2}}\{me^{-i\frac{\pi}{2}}(-Q)^{1/2}\}}{e^{-i\frac{\pi}{2}(\alpha+\frac{n-2}{2})}\{(-Q)^{1/2}\}^{\alpha+\frac{n-2}{2}}}+$ $+\theta(-Q)\theta(x_0)2^\alpha(2\pi)^{\frac{n-2}{2}}m^{\alpha+\frac{n-2}{2}}.$ $\cdot\frac{K_{\alpha+\frac{n-2}{2}}\{me^{i\frac{\pi}{2}}(-Q)^{1/2}\}}{e^{i\frac{\pi}{2}(\alpha+\frac{n-2}{2})}\{(-Q)^{1/2}\}^{\alpha+\frac{n-2}{2}}}+$ $+2^{2\alpha+\frac{n-4}{2}}(2\pi)^{\frac{n-2}{2}}i\pi\ sgn\ x_0\delta(Q)^{\alpha+\frac{n-4}{2}}$, if $2(\alpha+\frac{n-2}{2})$ is even, $\theta(Q)2^\alpha(2\pi)^{\frac{n-2}{2}}m^{\alpha+\frac{n-2}{2}}\frac{K_{\alpha+\frac{n-2}{2}}[mQ^{1/2}]}{[Q^{1/2}]^{\alpha+\frac{n-2}{2}}}+$ $+\theta(-Q)\theta(x_0)2^\alpha(2\pi)^{\frac{n-2}{2}}m^{\alpha+\frac{n-2}{2}}.$ $\cdot\frac{K_{\alpha+\frac{n-2}{2}}\{me^{-i\frac{\pi}{2}}(-Q)^{1/2}\}}{e^{-i\frac{\pi}{2}(\alpha+\frac{n-2}{2})}\{(-Q)^{1/2}\}^{\alpha+\frac{n-2}{2}}}+$ $+2^{2\alpha+\frac{n-4}{2}}(2\pi)^{\frac{n-2}{2}}\Gamma(\alpha+\frac{n-2}{2}).[\theta(-Q)(-Q)]^{-(\alpha+\frac{n-2}{2})}-$ $-e^{i\frac{\pi}{2}(\alpha+\frac{n-2}{2})}.\ sgn\ x_0[\theta(Q)Q]^{-(\alpha+\frac{n-2}{2})}$ if $2(\alpha+\frac{n-2}{2})$ is odd.

(99)	$G_A(t,\alpha,m^2,n)$	$\theta(Q)2^{\alpha}(2\pi)^{\frac{n-2}{2}}m^{\alpha+\frac{n-2}{2}}\frac{K_{\alpha+\frac{n-2}{2}}[mQ^{1/2}]}{[Q^{1/2}]^{\alpha+\frac{n-2}{2}}}+$ $+\theta(-Q)\theta(x_0)2^{\alpha}(2\pi)^{\frac{n-2}{2}}m^{\alpha+\frac{n-2}{2}}\frac{1}{2}i\pi e^{i\pi(\alpha+\frac{n-2}{2})}.$ $\frac{H^{(1)}_{\alpha+\frac{n-2}{2}}(m(-Q)^{1/2})}{[(-Q)^{1/2}]^{\alpha+\frac{n-2}{2}}}+$ $+\theta(-Q)\theta(-x_0)2^{\alpha}(2\pi)^{\frac{n-2}{2}}m^{\alpha+\frac{n-2}{2}}(-\frac{1}{2})i\pi e^{-i\pi(\alpha+\frac{n-2}{2})}.$ $\frac{H^{(2)}_{\alpha+\frac{n-2}{2}}(m(-Q)^{1/2})}{[(-Q)^{1/2}]^{\alpha+\frac{n-2}{2}}}+A(\alpha,n,Q),$ where $A(\alpha,n,Q)=2^{2\alpha+\frac{n-4}{2}}(2\pi)^{\frac{n-2}{2}}i\pi\ sgn\ x_0.$ $.\delta(Q)^{\alpha+\frac{n-4}{2}},$ if $2(\alpha+\frac{n-2}{2})$ is even, and $A(\alpha,n,Q)=2^{2\alpha+\frac{n-4}{2}}(2\pi)^{\frac{n-2}{2}}[\theta(-Q)(-Q)]^{-(\alpha+\frac{n-2}{2})}-$ $-e^{-i\pi(\alpha+\frac{n-2}{2})}\ sgn\ x_0[\theta(Q)Q]^{-\alpha+\frac{n-2}{2}},$ if $2(\alpha+\frac{n-2}{2})$ is odd.

Fourier Transforms in R^n

(100)	$W(t,\alpha,m^2,n) = \frac{(m^{-2}u)^{\frac{\alpha-n}{4}}}{\pi^{\frac{n-2}{2}} 2^{\frac{2\alpha+n-2}{2}} \Gamma(\frac{\alpha}{2})}$. $.J_{\frac{\alpha-n}{2}}\{\sqrt{m^2-u}\}$, if $t \in \Gamma_+$; $W(t,\alpha,m^2,n) = 0$ if $t \notin \Gamma_+$	$\theta(Q)(Q+m^2)^{-\alpha/2} + \theta(-Q)\theta(x_0).e^{-i\frac{\pi\alpha}{2}}(-Q+m^2)^{-\alpha/2} +$ $+\theta(-Q)\theta(-x_0).e^{i\frac{\pi\alpha}{2}}(-Q+m^2)^{-\alpha/2}$
(101)	$W(t,\alpha,m=0,n) = R_\alpha(u) =$ $= \{ \begin{matrix} \frac{u^{\frac{\alpha-n}{2}}}{H_n(\alpha)} & \text{if } t \in \Gamma_+, \\ 0 & \text{if } t \notin \Gamma_+. \end{matrix}$. ver p.35	$\theta(Q)Q^{-\alpha/2} + \theta(-Q)\theta(x_0).e^{-i\frac{\pi\alpha}{2}}(-Q)^{-\alpha/2} +$ $+\theta(-Q)\theta(-x_0).e^{i\frac{\pi\alpha}{2}}(-Q)^{-\alpha/2}$.
(102)	$G(t,\alpha,m^2,n) =$ $= G_R(t,\alpha,m^2,n) + G_A(t,\alpha,m^2,n)$	$2^\alpha(2\pi)^{\frac{n-2}{2}} m^{\alpha+\frac{n-2}{2}}$. $.\{\frac{K_{\alpha+\frac{n-2}{2}}[m(Q-i0)^{1/2}]}{(Q-i_0)^{1/2(\alpha+\frac{n-2}{2})}} + \frac{K_{\alpha+\frac{n-2}{2}}[m(Q+i0)^{1/2}]}{(Q+i_0)^{1/2(\alpha+\frac{n+2}{2})}}\}$
(103)	$G(t,\alpha=0,m^2,n) = \delta(u-m^2)$	$2^{\frac{n}{2}-1} m^{n-2-1} \pi^{\frac{n}{2}-1}$. $.\{\frac{K_{\frac{n}{2}-1}[m(Q-i_0)^{1/2}]}{[(Q-i_0)^{1/2}]^{n-2-1}} + \frac{K_{\frac{n}{2}-1}[m(Q+i_0)]^{1/2}}{(Q+i_0)^{1/2}]^{\frac{n}{2}-1}}\}$
(104)	$G(t,\alpha,m=0,n) = \frac{u^{\alpha-1}}{\Gamma(\alpha)}$controlar	$(2\pi)^{\frac{n-2}{2}} 2^{2\alpha+\frac{n-2}{2}-1}$. $.\{[(Q-i0)^{1/2}]^{-2\alpha-n+2} + [(Q+i_0)^{1/2}]^{-2\alpha-n-2}\}$
(105)	$\delta_R^{(k)}(u)$, $k = \frac{n-2}{2} + h, h = 0,1,\ldots$	$-i(2\pi)^{\frac{n-2}{2}} 2^{\frac{n}{2}-2h} \pi sgn x_0 \frac{Q}{\Gamma(h+1)} +$ $+\frac{\pi^{\frac{n-2}{2}}}{2^{1-2h}\Gamma(h+1)}\{Q^h[\Psi(h+1) + \Psi(1) + 2lg2 - lg\|Q\|]\}$
(106)	$W(t,\alpha,m=0,n) = R_\alpha(u) =$ $= \{ \begin{matrix} \frac{u^{\frac{\alpha-n}{2}}}{H_n(\alpha)} & \text{if } t \in \Gamma_+ \\ 0 & \text{if } t \notin \Gamma_+ \end{matrix}$. $H_n(\alpha) = \pi^{\frac{n-2}{2}} 2^{\alpha-1}\Gamma(\frac{\alpha}{2})\Gamma(\frac{\alpha-n+2}{2})$, here α is an exceptional value	$\{x_1^2 + x_2^2 + \cdots + x_{n-1}^2 - (x_0 - i_0)^2\}^{-\alpha/2}$

(107)	$W(t,\alpha,m\neq 0,n)$, here α is an exceptional value. $Re\ \alpha \leq 0$	$\theta(Q)(m^2+Q)^{-\alpha/2}+\theta(-Q)\theta(x_0)\{(m^2+Q)_+^{-\alpha/2}+e^{-i\frac{\pi\alpha}{2}}(m^2+Q)_-^{-\alpha/2}\}+$ $+\theta(-Q)\theta(-x_0)\{(m^2+Q)_+^{-\alpha/2}+$ $+e^{i\frac{\pi\alpha}{2}}(m^2+Q)_-^{-\alpha/2}\}$
(108)	$W(t,\alpha,m\neq 0,n)$, $\alpha=2k,\quad k=1,\ldots$	$(m^2+Q)^{-k}+\frac{(-1)^k}{(k-1)!}sgn x_0\delta^{(k-1)}(m^2+Q)$
(109)	$G_R(t,\alpha>0,m^2,n)$, in the singular points, n even	see ([14], $\oint$ XII)
(110)	$G_R(t,\alpha,m=0,n)$, in the singular points, $2\alpha-2+n$ is even	see ([14], $\oint$ XIII.1)
(111)	$G_R(t,\alpha,m=0,n)$, in the singular points, $2\alpha-2+n$ is odd	see ([14], $\oint$ XIII.2)
(112)	$K^l(\delta)*\delta^{(k)}(m^2+P)_+$	$\frac{(-1)^l 2^{-k} m^{\frac{1}{2}-k-1}}{(2\pi i)(2\pi)^n}.\{e^{-i\frac{\pi}{2}q}.(m^2+Q-i0)^l\frac{K_{\frac{n}{2}-k-1}[m(Q-i0)^{1/2}]}{(Q-i_0)^{\frac{1}{2}(\frac{n}{2}-k-1)}}-$ $-e^{i\frac{\pi}{2}q}.(m^2+Q-i_0)^l\frac{K_{\frac{n}{2}-k-1}[m(Q+i_0)^{1/2}]}{(Q+i_0)^{\frac{1}{2}(\frac{n}{2}-k-1)}}\}.$
(113)	$(m^2+P)^l*\delta^{(k)}(m^2+P)_+$	$(-1)^l\frac{\Gamma(k+1)}{2\pi!}\{G_{-2k}(Q-i0,m,n).G_{2+2k}(Q-i_0,m,n)-$ $-G_{-2k}(Q+i_0,m,n).G_{2+2k}(Q+i_0,m,n)\}$. see [22].

(114) $(m^2+P)^l.\delta^{(k)}(m^2+P) =$	
$\frac{(-1)k!}{(k-l)!}\delta^{k-l}(m^2+P)$	$\frac{(-1)^l k!}{2\pi i}[G_{2k+2-2l}(Q-i_0,m,n)-G_{2k+2-2l}(Q+i_0,m,n)]$
$k,l \in Z^+, \quad k \geq l.$	
$\{P^l.\delta^{(k)}(P)\} =$ $k,l \in Z^+$ $k < \frac{n}{2}-1,$ (115) $k-l < \frac{n}{2}-l, k \leq l$	$\frac{(-1)^l k!}{(2\pi i)}[G_{2k+2-2l}(Q-i0,m=0,n)-G_{2k+2-2l}(Q+i0,m=0,n)] =$ $= \frac{(-1)^l k!}{(2\pi i)}[H_{2k+2-2l}(Q-i0,n)-H_{2k+2-2l}(Q+i0,n)]$

Fourier Transforms (Bremermann)

	$f(x)$	$\mathcal{F}[f(x)] = \int_{E^n} f(x) e^{i<x,w>} dx$
(116)	$X(t) e^{i<t,z>}$	$\frac{i^{n+1}\Gamma(\frac{n+1}{2})}{2\pi^{\frac{n+3}{2}}[(\xi - z)^2]^{\frac{n+1}{2}}}$

Here $X(t)$ is the characteristic function of the forward light cone $\Gamma^+ = \{x \in R^{n+1}/x^2 > 0, x_0 > 0\}$

$x^2 = x_0^2 - x_1^2 - \cdots - x_n^2, \qquad (x_0, x_1, \ldots, x_{n-1}) \in R^{n+1},$

$T^+ = \{z/y \in \Gamma^+, x \in R^{n+1},$

$z \in C^{n+1}, z_j = x_j + iy_j; \quad j = 0, 1, \ldots, n-1.$

Fourier Transforms (Constantinescu)

	$f(x)$	$\mathcal{F}[f(x)] = \int_{R^n} f(x) e^{i\xi x} dx$
(117)	1	$(2\pi)^n \delta$
(118)	δ	1
(119)	e^{ihx}	$(2\pi)^n \delta_{-h}$
(120)	δ_{-h}	$e^{ih\xi}$
(121)	$D^\alpha \delta$	$(-i\xi)^\alpha$
(122)	r^λ	$2^{\lambda+n} \pi^{\frac{n}{2}} \frac{\Gamma(\frac{\lambda+n}{2})}{\Gamma(-\frac{\lambda}{2})} \lvert\xi\rvert^{-n-\lambda}$

Fourier Transforms (Gelfand - Shilov)

	$f(x)$	$\mathcal{F}[f(x)] = \int_R^\infty f(x)e^{ix\sigma}dx$
(123)	$\delta(x_1, x_2, \ldots, x_n)$	1
(124)	1	$(2\pi)^n\delta(\sigma_1, \sigma_2, \ldots \sigma_n)$
(125)	Polynomial $P(x_1, x_2, \ldots, x_n)$	$(2\pi)^n P(-i\frac{\partial}{\partial\sigma_1}, \ldots, -i\frac{\partial}{\partial\sigma_n})\delta(\sigma)$
(126)	$r^\lambda (r = \sqrt{\sum_{i=1} x_j^2})$	$2^{\lambda+n}\pi^{\frac{n}{2}}\frac{\Gamma(\frac{\lambda+n}{2})}{\Gamma(-\frac{\lambda}{2})}\rho^{-\lambda-n}(\rho = \sqrt{\sigma_j^2})$
(127)	$f_\lambda(r) = \frac{2^{-\frac{1}{2}\lambda}r^\lambda}{\Gamma(\frac{\lambda+n}{2})}$	$(2\pi)^{\frac{n}{2}}f_{-\lambda-n}(\rho) = (2\pi)^{\frac{1}{2}n}\frac{2^{\frac{1}{2}(\lambda+n)}r^{-\lambda-n}}{\Gamma(-\frac{\lambda}{2})}$
(128)	$r^\lambda\, ln\, r$	$d\frac{C_\lambda}{d_\lambda}\rho^{-\lambda-n} + C_\lambda\rho^{\lambda-n}\, ln\, \rho$ (1) $(\lambda \neq -n, -n-2, \ldots)$
(129)	$r^\lambda\, ln^2\, r$	$\frac{d^2C_\lambda}{d\lambda^2}\rho^{-\lambda-n} + 2\frac{dC_\lambda}{d\lambda}\rho^{-\lambda-n}\, ln\, \rho + C_\lambda\rho^{-\lambda-n}\, ln^2\, \rho$ $(\lambda \neq -n, -n-2, \ldots)$
(130)	$\Omega_n r^{-2m-n}$	$c_{-1}^{(n+2m)}\rho^{2m}\, ln\, \rho + c_0^{(n+2m)}\rho^{2m}$ (2)
(131)	$\Omega_n r^{-2m-n}\, ln\, r$	$\frac{1}{2}c_{-1}^{(n+2m)}\rho^{2m}\, ln^2\, \rho + c_0^{(n+2m)}\rho^{2m}\, ln\, \rho + c_1^{(n+2m)}\rho^{2m}$
(132)	$\delta(r-a)$	$2^{\frac{n}{2}-1}\Gamma(\frac{n}{2}-\frac{1}{2})\Gamma(\frac{1}{2})\Omega_{n-1}a^{\frac{1}{2}n}\rho^{1-\frac{1}{2}n}J_{\frac{1}{2}(n-2)}(a\rho)$
(133)	$(\frac{d}{ada})^m\frac{\delta(r-a)}{a}$	$2^{\frac{n}{2}-1}\Gamma(\frac{n}{2}-\frac{1}{2})\Gamma(\frac{1}{2})\sqrt{\frac{a}{\pi}}\Omega_{n-1}\sqrt{\frac{2}{\pi}}\frac{sin\, a\rho}{\rho}$

(1) $C_\lambda = \frac{2^{\lambda+n}\pi^{\frac{n}{2}}\Gamma(\frac{\lambda+n}{2})}{\Gamma(-\frac{\lambda}{2})} = \frac{C_{-1}^{(n+2m)}}{\lambda+n+2m} + c_0^{(n+2m)} + c_1^{(n+2m)}(\lambda+n+2m) + \ldots$

(2) $\Omega_n = \frac{2\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2})}$

(134)	$(P+i0)^{\lambda}$	$\frac{1}{\Gamma(-\lambda)}e^{-i\frac{\pi}{2}qi}2^{n+2\lambda}\pi^{\frac{n}{2}}\Gamma(\lambda+\frac{n}{2})(Q-i0)^{-\lambda-\frac{n}{2}}$
(135)	$(P-i0)^{\lambda}$	$\frac{1}{\Gamma(-\lambda)}e^{i\frac{\pi}{2}qi}2^{n+2\lambda}\pi^{\frac{n}{2}}\Gamma(\lambda+\frac{n}{2})(Q+i0)^{-\lambda-\frac{n}{2}}$
(136)	P_{+}^{λ}	$2^{n+2\lambda}\pi^{\frac{n}{2}-1}\Gamma(\lambda+1)\Gamma(\lambda+\frac{n}{2}).\frac{1}{2i}$ $[e^{-i(\frac{q}{2}+\lambda)\pi}(Q-i0)^{-\lambda-\frac{n}{2}}-e^{-i(\frac{q}{2}+\lambda)\pi}(Q+i0)^{-\lambda-\frac{n}{2}}]$
(137)	$(P\pm i0)^{h}lg(P\pm i0)$	$\frac{\Gamma(\frac{n}{2}+h)\Gamma(h+1)(-1)^{h+1}}{\pi^{-\frac{n}{2}}2^{-n-2h}}.Pf(Q\mp io)^{-\frac{1}{2}(n+2h)}+(-1)\{2\,lg\,2+\frac{\Gamma'(h+1)}{\Gamma(h+1)}+$ $+\frac{\Gamma'(\frac{n+2h}{2})}{\Gamma(\frac{n+2h}{2})}\}.\{\frac{2^{n+2h}\pi^{n}\Gamma(\frac{n+2h}{2})}{\pi^{n/2}\Gamma(-h)}e^{\mp i\ \pi/2\ qi}\}.(Q\mp i0)^{1/2\ (n+2h)}.$
(138)	P_{-}^{λ}	$-2^{n+2\lambda}\pi^{\frac{n}{2}-1}\Gamma(\lambda+1)\Gamma(\lambda+\frac{n}{2}).$ $.\frac{1}{2i}[e^{-\frac{\pi}{2}qi}(Q-i0)^{-\lambda-\frac{n}{2}}-e^{\frac{\pi}{2}qi}(Q+i0)^{-\lambda-\frac{n}{2}}]$
(139)	$(m^2+Q+i0)^{\lambda}$	$\frac{2^{\lambda+1}(\sqrt{2\pi})^{n}m^{\frac{1}{2}n+\lambda}}{\Gamma(-\lambda)}\frac{K_{\frac{n}{2}+\lambda}[m(Q-i0)^{\frac{1}{2}}]}{(Q-i0)^{\frac{1}{2}(\frac{1}{2}n+\lambda)}}=\frac{2^{\lambda+\frac{n}{2}+1}\pi^{\frac{n}{2}}e^{-\frac{1}{2}q\pi}m^{\lambda+\frac{n}{2}}}{\Gamma(-\lambda)}.$ $[\frac{K_{\lambda+\frac{n}{2}}(mQ_{+}^{1/2})}{Q_{+}^{1/2(\lambda+n/2)}}+\frac{\pi i}{2}\frac{H^{(1)}_{-\lambda-n/2}(mQ_{-}^{1/2})}{Q_{-}^{1/2(\lambda+n/2)}}]$
(140)	$(m^2+Q-i0)^{\lambda}$	$\frac{2^{\lambda+1}(\sqrt{2\pi})^{n}m^{\frac{1}{2}n+\lambda}}{\Gamma(-\lambda)}\frac{K_{\frac{n}{2}+\lambda}[m(Q+i0)^{\frac{1}{2}}]}{(Q+i0)^{\frac{1}{2}(\frac{1}{2}n+\lambda)}}=\frac{2^{\lambda+\frac{n}{2}+1}\pi^{\frac{n}{2}}e^{\frac{iq\pi}{2}}m^{\lambda+\frac{n}{2}}}{\Gamma(-\lambda)}.$ $[\frac{K_{\lambda+n/2}(mQ_{+}^{1/2}}{Q_{+}^{1/2}(\lambda+\frac{1}{2}n)}-\frac{\pi i}{2}\frac{H^{(2)}_{-\lambda-1/2n}(mQ_{-}^{1/2})}{Q_{-}^{1/2(\lambda+n/2)}}]$
(141)	$\frac{(m^2+P)_{\pm}^{\lambda}}{\Gamma(\lambda+1)}$	$2^{\lambda+n/2}i\pi^{n/2\ -1}m^{n/2\ \lambda}.\{e^{-i(\lambda+q/2)\pi}\frac{K_{n/2\ +\lambda}[m(Q-i0)^{1/2}]}{(Q-i0)^{1/2}(\lambda+n/2)}-$ $-e^{i(\lambda+\frac{q}{2}\pi}\frac{K_{n/2\ +\lambda}[m(Q+i0)^{1/2}]}{(Q+i0)^{1/2}(\lambda+n/2)}\}=$ $=2^{\lambda+\frac{n}{2}+1}\pi^{\frac{n}{2}-1}m^{\frac{n}{2}+\lambda}.\{\frac{-sin(\lambda+\frac{q}{2})\pi K_{\lambda+\frac{n}{2}}(mQ_{+}^{1/2})}{Q_{+}^{1/2}(\lambda+n/2)}+\frac{\pi}{2\,sin(\lambda+\frac{n}{2})\pi}$ $[sin(\lambda+\frac{q}{2})\pi\frac{J_{\lambda+n/2}(mQ_{-}^{1/2})}{Q_{-}^{\frac{1}{2}(\lambda+\frac{n}{2})}}+sin\,\frac{p\pi}{2}\frac{J_{-\lambda-n/2}(mQ_{-}^{1/2})}{Q_{-}^{\frac{1}{2}(\lambda+\frac{n}{2})}}\}$

Fourier Transforms (Gelfand - Shilov)

(142)	$\frac{(m^2+P)_-^\lambda}{\Gamma(\lambda+1)}$	$2^{\lambda+\frac{n}{2}} i\pi^{\frac{n}{2}-1} m^{\frac{n}{2}+\lambda}\{\frac{e^{-i\frac{\pi}{2}q}K_{\frac{n}{2}+\lambda}[m(Q-i0)^{1/2}]}{(Q-i0)^{1/2(\lambda+\frac{n}{2})}} - \frac{e^{i\frac{\pi}{2}qi}K_{\frac{n}{2}+\lambda}[m(Q+i0)^{1/2}]}{(Q+i0)^{\frac{1}{2}(\lambda+\frac{n}{2})}}\} =$ $= 2^{\lambda+\frac{n}{2}+1}\pi^{\frac{n}{2}-1}m^{\frac{n}{2}+\lambda}.\{sin\,\frac{q\pi}{2}\frac{K_{\lambda+\frac{n}{2}}(mQ_+^{1/2})}{Q_+^{\frac{1}{2}(\lambda+\frac{n}{2})}} - \frac{\pi}{2\,sin(\lambda+\frac{1}{2}n)\pi}.$ $.[sin\,\frac{q\pi}{2}\frac{J_{\lambda+\frac{n}{2}}(mQ_-^{1/2})}{Q_-^{\frac{1}{2}(\lambda+\frac{n}{2})}} + \frac{sin(\lambda+\frac{p}{2})\pi J_{-\lambda-\frac{n}{2}}(mQ_-^{1/2})}{Q_-^{\frac{1}{2}(\lambda+\frac{1}{2}n)}}]\}$
(143)	$\delta^{(k)}(m^2+P)$	$\frac{k!}{2\pi i}\{G_{2k+2}(P-i0,m,n) - G_{2k+2}(P+i0,m,n)\}$
(144)	$\delta^{(t-1)}(m^2+P)$	$(-1)^{t+1}i2^{\frac{n}{2}-t}\pi^{\frac{n}{2}-1}.[e^{-i\frac{\pi}{2}q}\frac{K_{\frac{n}{2}-t}[m(Q-i0)^{\frac{1}{2}}]}{(Q-i0)^{\frac{1}{2}(\frac{n}{2}-q)}} - e^{i\frac{\pi}{2}q}\frac{K_{\frac{n}{2}-q}[m(Q+i0)^{\frac{1}{2}}]}{(Q+i0)^{\frac{1}{2}(\frac{n}{2}-q)}}]$
(145)	$\delta(m^2+P)$	$-i(2\pi m)^{\frac{n}{2}-1}[-e^{-i\frac{\pi}{2}q}\frac{K_{\frac{n}{2}-1}[m(Q-i0)^{\frac{1}{2}}]}{(Q-i0)^{\frac{1}{2}(\frac{n}{2}-1)}}] + e^{i\frac{\pi}{2}q}\frac{K_{\frac{n}{2}-1}[m(Q+i0)^{\frac{1}{2}}]}{(Q+i0)^{\frac{1}{2}(\frac{n}{2}-1)}}]$
(146)	$\frac{(m^2+P)_+^t}{\Gamma(t+1)}$	$(-1)^{t+1}i2^{t+\frac{n}{2}}\pi^{\frac{n}{2}+t}.[e^{-\frac{i}{2}\pi q}\frac{K_{\frac{n}{2}+t}[m(Q-i0)^{\frac{1}{2}}]}{(Q-i0)^{\frac{1}{2}(\frac{n}{2}+q)}}]-$ $-e^{i\frac{\pi}{2}g}\frac{K_{\frac{n}{2}+t}[m(Q+i0)^{\frac{1}{2}}]}{(Q+i0)^{\frac{1}{2}(\frac{n}{2}+q)}}] + (2\pi)^n\sum_{m=0}^{t}\frac{(-1)^m(\frac{m}{2})^{2t-2m}}{4^m m!(t-m)!}L^m\delta(s)$
(147)	$\frac{(m^2+P)_-^t}{\Gamma(t+1)}$	$i2^{t+\frac{n}{2}}\pi^{\frac{n}{2}-1}m^{\frac{n}{2}+t}.[e^{-i\frac{\pi}{2}q}\frac{K_{\frac{n}{2}+t}[m(Q-i0)^{\frac{1}{2}}]}{(Q-i0)^{\frac{1}{2}(\frac{n}{2}+q)}}] - e^{\frac{\pi}{2}qi}\frac{K_{\frac{n}{2}+t}[m(Q+i0)^{\frac{1}{2}}]}{(Q+i0)^{\frac{1}{2}(\frac{n}{2}+q)}}]$
(148)	$\frac{(m^2+P)^t}{\Gamma(t+1)}$	$(2\pi)^n\sum_{p=0}^{t}\frac{(-1)^p(\frac{m}{2})^{2t-2p}}{4^p p!(t-p)!}L^p\delta(s)$

Fourier Transforms (Jones)

	$f(x)$	$F[f(x)] = \int_{R^n} e^{-i<x,y>} f(x) dx$
(149)	$\int_{-\infty}^{\infty} \delta(x) e^{-i\alpha x} dx$	1
(150)	$f(x) = g(r^2)$	$\frac{(2\pi)^{(\frac{n}{2})}}{\rho^{\frac{n-2}{2}}} \int_0^{\infty} g(t^2) t^{\frac{n}{2}} J_{\frac{n-2}{2}}(\rho t) dt$

Fourier Transforms (Schwartz)

	$f(x)$	$\mathcal{F}[f(x)] = \int_{R^n} e^{2i\pi xy} f(x)dx$
(151)	r^{2h}	$\{-\frac{\Delta}{4\pi^2}\}^h \delta$
(152)	$Pf\frac{1}{r^{n+2h}}$	$\frac{\pi^{\frac{n}{2}+2h}}{\Gamma(\frac{n}{2}h)} 2\frac{(-1)^h}{h!} r^{2h}[log\frac{1}{\pi r} + \frac{1}{2}(1 + \frac{1}{2} + \cdots + \frac{1}{h} - \mathcal{C}) + \frac{1}{2}\frac{\Gamma'(\frac{n}{2}+h)}{\Gamma(\frac{n}{2}+h)}](*)$
(153)	$Pf\frac{1}{r^n}$	$\frac{2(\sqrt{\pi})^n}{\Gamma(\frac{n}{2})}[log\frac{1}{\pi r} - \frac{\mathcal{C}}{2} + \frac{1}{2}\frac{\Gamma'(\frac{n}{2})}{\Gamma(\frac{n}{2})}]$
(154)	$lg\frac{1}{r}$	$\frac{\Gamma(\frac{n}{2})}{2(\sqrt{\pi})^n}(Pf\frac{1}{r^n}) + (\frac{\mathcal{C}}{2} - \frac{1}{2}\frac{\Gamma'(\frac{n}{2})}{\Gamma(\frac{n}{2})} + log\,\pi)\delta$
(155)	$\Delta(\frac{1}{r^{n-2}})$	$-(n-2)\frac{2\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2})}$, $\qquad n \neq 2$
(156)	$\Delta[lg(\frac{1}{r})]$	-2π, $\qquad n = 2$
(157)	$lg\|x\|$	$-\frac{1}{2}Pf(\frac{1}{\|y\|}) - (\mathcal{C} + log\,2\pi)\delta$, $\qquad n = 1$
(158)	$(1+r^2)^{-\frac{m}{2}}$	$\frac{2\pi^{\frac{m}{2}}}{\Gamma(\frac{m}{2})} r^{m-n} K_{\frac{n-m}{2}}(2\pi r)$
(159)	$Z_l e^{-2\pi\varepsilon x_n}$	$(\frac{1}{2\pi})^l[(\varepsilon + iy_n)^2 + y_1^2 + y_2^2 + \cdots + y_{n-1}^2]^{-\frac{1}{2}}$ (**)
(160)	Z_{2k}	$Pf(\frac{-1}{4\pi^2\sigma^2})^k$, $\qquad \sigma^2 = y_n^2 - y_1^2 - y_2^2 - \cdots - y_{n-1}^2$
(161)	$e^{-\pi r^2}$	$e^{-\pi\rho^2}$

(*) where $\mathcal{C}$ is the Euler constant; and the sum $1 + \frac{1}{2} + \cdots + \frac{1}{h}$ is zero if $h = 0$.

(**) $$Z_l = \frac{1}{\pi^{\frac{n-2}{2}} 2^{l-1}\Gamma(\frac{l}{2})\Gamma(\frac{l+2-n}{2})} Pf(s^{l-n})$$

$s = (x_n^2 - x_1^2 - x_2^2 - \cdots - x_{n-1}^2)^{\frac{1}{2}}, \qquad x_n \geq 0.$

Fourier Transforms (Vladimirov)

	$f(x)$	$\mathcal{F}[f(x)] = \int_{R^n} f(\xi) e^{i\xi x} d\xi$
(162)	$[\lvert x\rvert^2 - (x_0 \pm i0)^2]^{-k}$	$\frac{\pm i\pi}{(k-1)!}\varepsilon(x_0)\delta^{(k-1)}(x^2) + (-1)^k P\frac{1}{(x^2)^k}$
(163)	$[\lvert x\rvert^2 - (x_0 \pm i0)^2]^{-k-\frac{1}{2}}$	$[-\theta(-x^2)x^2]^{-k-\frac{1}{2}} \pm i(-1)^k \varepsilon(x_0)[\theta(x^2)x^2]^{-k-\frac{1}{2}}$

	$f(x)$	$\mathcal{H}[f(x)] = \int_{R^n} f(x) J_\nu(xy)\sqrt{xy}dx$	
(1)	$\lvert x\rvert^{\mu-\frac{n}{2}+\frac{1}{2}}$	$\frac{2^{\mu+\frac{1}{2}}\lvert y\rvert^{-\mu-\frac{n}{2}-\frac{1}{2}}\Gamma(\frac{\mu}{2}+\frac{n}{4}+\frac{1}{4})}{\Gamma(\frac{n}{2}-\frac{\mu}{2})}$,	$-\frac{n}{2}-\frac{1}{2} < Re\mu < -\frac{1}{2}$.
(2)	$1 \quad 0 < \sqrt{x_1^2+\cdots+x_n^2} < 1$ $0 \quad 1 < \sqrt{x_1^2+\cdots+x_n^2} < \infty$	$\frac{1}{\lvert y\rvert^{\frac{n}{2}}} J_{\frac{n}{2}}(\lvert y\rvert)$	$\frac{n-2}{2} > -1$
(3)	$\lvert x\rvert^{\frac{(2-n)}{2}} \quad 0 < \lvert x\rvert^{\frac{1}{2}} < 1$ $0 \quad 1 < \lvert x\rvert^{\frac{1}{2}} < \infty$	$\frac{2^{\frac{2-n}{2}}\lvert y\rvert^{-2}}{\Gamma(\frac{n}{2}-1)} - \lvert y\rvert^{-\frac{1}{2}} J_{\frac{n}{2}-2}(\lvert y\rvert)$	$n \neq 2$
(4)	$\lvert x\rvert^{-\frac{n}{2}}(\lvert x\rvert^2+a^2)^{\frac{-1}{2}}$	$\lvert y\rvert^{1-\frac{n}{2}} I_{\frac{n-2}{2}}(\frac{a\lvert y\rvert}{2}) K_{\frac{n-2}{2}}(\frac{a\lvert y\rvert}{2})$	$\frac{n-2}{2} > -1$
(5)	$(\lvert x\rvert^2+a)^{-1}$	$\frac{a^{\frac{n-2}{2}}}{\lvert y\rvert^{\frac{n-2}{2}}} K_{\frac{n-2}{2}}(a\lvert y\rvert)$	
(6)	$(\lvert x\rvert^2+a^2)^{-\frac{1}{2}}$	$2^{1/2}\pi^{-1/2} a^{\frac{n-1}{2}} K_{\frac{n-1}{2}}(ay)$	
(7)	$(\lvert x\rvert^2+a^2)^{-\frac{n-1}{2}}$	$\frac{1}{\lvert y\rvert} \frac{\pi^{1/2}}{2^{\frac{n-2}{2}}\Gamma(\frac{n-1}{2})} \frac{1}{e^{a\lvert y\rvert}}$	$n > 1$
(8)	$(\lvert x\rvert^2+a^2)^{-\mu-1}$	$\frac{a^{\frac{n-2}{2}-\mu}}{2^\mu\Gamma(\mu+1)} \lvert y\rvert^{\mu-\frac{n}{2}+1} K_{\frac{n-2}{2}-\mu}(a\lvert y\rvert)$	
(9)	$0 \quad 0 < r < a$ $r^{2-n}(r^2-a^2)^{-\mu-1} \quad a < r < \infty$	$2^\mu\Gamma(\mu+1) a^{1+\mu-\frac{n}{2}+1} \; \lvert y\rvert^{-\mu-\frac{1}{2}} \; J_{\frac{n-2}{2}-\mu-1}$	

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(10)	$(a^2-\lvert x\rvert^2)^\mu \quad 0<\lvert x\rvert<a$ $0 \quad a<\lvert x\rvert<\infty$	$\lvert y\rvert^{-\mu-\frac{n}{2}}2^\mu a^{\mu+\frac{n}{2}}J_{\mu+\frac{n}{2}}(a\lvert y\rvert)$	
(11)	$\lvert x\rvert^{-\frac{n}{2}}(a^2-\lvert x\rvert^2)^\lambda$	$a^{2\lambda+\mu+\frac{n}{2}}B(\lambda+1,\frac{\mu}{2}+\frac{n-1}{2}).(2^{\frac{n}{2}-1}\Gamma(\frac{n}{2}-1))^{-1}.$ $\cdot\ {}_1F_2(\frac{\mu+\frac{n}{2}}{2},\frac{n}{2},\frac{2+\mu+\frac{n}{2}}{2}+\lambda;-\frac{\lambda^2}{4}\lvert y\rvert^2)$	$0<\lvert x\rvert<a$
(12)	$\lvert x\rvert^{-\frac{n}{2}}(a^2+\lvert x\rvert^2)[(a^2+\lvert x\rvert^2)^{1/2}+\lvert x\rvert]^{\frac{n}{2}-2}$	$\lvert y\rvert^{-\frac{n-1}{2}}\pi^{1/2}e^{-a\frac{\lvert y\rvert}{2}}I_{\frac{n-3}{2}}(\frac{a\lvert y\rvert}{2})$	
(13)	$\frac{[(\lvert x\rvert^2+a^2)^{1/2}-a]^{\frac{n-2}{2}}}{\lvert x\rvert^{n-2}(\lvert x\rvert^2+a)^{1/2}}$	$\lvert y\rvert^{-\frac{n}{2}}e^{-a\lvert y\rvert}$	
(14)	$\lvert x\rvert^2(\lvert x\rvert^4+4a^4)^{-\frac{n}{2}+\frac{1}{2}}$	$\frac{\pi^{1/2}J_{\frac{n-2}{2}}(a\lvert y\rvert)K_{\frac{n-2}{2}}(a\lvert y\rvert)}{2^{\frac{3}{2}n-4}a^{n-4}\Gamma(\frac{n-1}{2})}$	$n\geq 3$
(15)	$(\lvert x\rvert^4+4a^4)^{-\frac{n}{2}+\frac{1}{2}}$	$\frac{\pi^{1/2}}{2^{3(\frac{n-2}{2})}a^{n-2}\Gamma(\frac{n-1}{2})}J_{\frac{n}{2}-1}(a\lvert y\rvert)K_{\frac{n-2}{2}}(a\lvert y\rvert)$	$n\geq 2$
(16)	$\lvert x\rvert^{-\frac{n}{2}}e^{-a\lvert x\rvert}$	$\lvert y\rvert^{-n+2}(\lvert y\rvert^2+a^2)^{-1/2}[(a^2+\lvert y\rvert^2)^{1/2}-a]^{\frac{n-2}{2}}$	
(17)	$\lvert x\rvert^{-\frac{n}{2}-1}e^{-a\lvert x\rvert}$	$\lvert y\rvert^{-n+2}\frac{2}{n-2}[(a^2+\lvert y\rvert^2)^{1/2}-a]^{\frac{n-2}{2}}$	$n\geq 3, a>0$
(18)	$\lvert x\rvert^{m-\frac{n}{2}-1}e^{-a\lvert x\rvert}$	$(-1)^{n+1}\lvert y\rvert^{-n+2}\frac{d^{n+1}}{da^{n+1}}\{(a^2+\lvert y\rvert^2)^{-1/2}[(a^2+\lvert y\rvert^2)^{1/2}-a]^{\frac{n-2}{2}}\}$	
(19)	$e^{-a\lvert x\rvert}$	$\pi^{-1/2}2^{n/2}\Gamma(\frac{n+1}{2})\frac{a}{(a^2+\lvert y\rvert^2)^{\frac{n+1}{2}}}$	
(20)	$\lvert x\rvert^{-1}e^{-a\lvert x\rvert}$	$\frac{1}{2}\frac{1}{\pi^{\frac{n+1}{2}}}\Gamma(\frac{n-1}{2})\frac{1}{(a^2+\lvert y\rvert)^{\frac{n-1}{2}}}$	

(21)	$\|x\|^{\mu-\frac{n}{2}-1}e^{-a\|x\|}$	$\frac{1}{\|y\|^{\frac{n-1}{2}}}\|y\|^{1/2}(a^2+\|y\|^2)^{-\mu/2}\Gamma(\mu+\frac{n}{2}-1).$ $.P_{\mu-1}^{-(\frac{n-2}{2})}[a(a^2+\|y\|^2)^{-1/2}] =$ $= \frac{\Gamma(\mu+\frac{n}{2}-1)}{2^{\frac{n}{2}-1}a^{\mu+\frac{n}{2}-1}\Gamma(\frac{n}{2})}.\ {}_2F_1(\frac{\mu+\frac{n}{2}-1}{2}, \frac{\mu+n}{2}, \frac{n}{2}; \frac{\|y\|^2}{a^2})$	
(22)	$\|x\|^{-n/2}e^{-a\|x\|^2}$	$\frac{\pi^{1/2}}{2a^{1/2}}\frac{1}{\|y\|^{\frac{n-2}{2}}}e^{-\frac{\|y\|^2}{8a}}I_{\frac{n-2}{2}}(\frac{\|y\|^2}{8a}) =$ $= \frac{\pi^{1/2}}{8a^{3/2}}\frac{1}{\|y\|^{\frac{n-4}{2}}}e^{-\frac{\|y\|^2}{8a}}.[I_{\frac{n-3}{2}}(\frac{\|y\|^2}{8a}) - I_{n/2}(\frac{\|y\|^2}{8a})]$	
(23)	$e^{-a\|x\|^2}$	$(4\pi a)^{-n/2}e^{-\frac{\|y\|^2}{4a}}$	
(24)	$\|x\|^{2k}e^{-\frac{\|x\|^2}{4}}$	$2^{2k+\frac{n}{2}}\ k!\ e^{-\|y\|^2}L_k^{\frac{n-2}{2}}(\|y\|^2)$	
(25)	$\|x\|^{\mu-\frac{n}{2}}e^{-a\|x\|^2}$	$\frac{\Gamma(\frac{n-2}{4}+\frac{\mu}{2}+\frac{1}{2})}{\|y\|^{n/2}a^{\mu/2}\Gamma(\frac{n}{2})}e^{-\frac{\|y\|^2}{8a}}M_{\frac{\mu}{2},\frac{n-2}{4}}(\frac{\|y\|^2}{4a})$	
(26)	$\|x\|^{-\frac{n}{2}-1}e^{-\frac{\alpha}{\|x\|}}$	$\frac{2}{\|y\|^{\frac{n-2}{2}}}J_{\frac{n-2}{2}}(2\alpha\|y\|^{1/2})K_{\frac{n-2}{2}}(2\alpha\|y\|^{1/2})$	
(27)	$\|x\|^{\mu-\frac{n}{2}+\frac{1}{2}}\ log\ \|x\|$	$\frac{1}{\|y\|^{\frac{n-1}{2}}}\frac{1}{\|y\|^{\mu+1}}\frac{2^{\mu-\frac{1}{2}}\Gamma(\frac{\mu}{2}+\frac{n-2}{4}+\frac{3}{4})}{\Gamma(\frac{n-2}{4}-\frac{\mu}{2}+\frac{1}{4})}.$ $.[\Psi(\frac{\mu}{2}+\frac{n-2}{4}+\frac{3}{4})+\psi(\frac{n-2}{4}-\frac{\mu}{2}+\frac{1}{4}) - log(\frac{1}{4}\|y\|^2)]$	
(28)	$\|x\|^{1-\frac{n}{2}}e^{-\beta\|x\|^2}I_{\frac{n-2}{2}}(a\|x\|)$	$\frac{1}{\|y\|^{\frac{n-2}{2}}}\frac{1}{2\beta}e^{\frac{(a^2-\|y\|^2)}{4\beta}}J_{\frac{n-2}{2}}(\frac{a\|y\|}{2\beta})$	$\frac{n-2}{2} > -1, \quad Re\ \beta > 0$
(29)	$\|x\|^{1-\frac{n}{2}}K_{\frac{n-2}{2}}(\alpha\|x\|)$	$\frac{1}{\alpha^{\frac{n-2}{2}}(\|y\|^2+\alpha^2)}$	$\frac{n-2}{2} > -1$

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(30)	$\lvert x\rvert^{\mu} K_{\mu}(\alpha\lvert x\rvert)$	$2^{\mu+\frac{n}{2}-1}\Gamma(\mu+\frac{n}{2})\alpha^{\mu}(\lvert y\rvert^2+\alpha^2)^{-\mu-\frac{n}{2}}$	$\frac{n}{2} > \lvert Re\ \mu\rvert$
(31)	$J_{\frac{n-2}{2}}(\frac{1}{2}a\lvert x\rvert)K_{\frac{n-2}{2}}(\frac{1}{2}a\lvert x\rvert)$	$\frac{a^{n-2}2^{\frac{n-2}{2}}\Gamma(\frac{n-1}{2})}{\Gamma(\frac{1}{2})}(\lvert y\rvert^4+a^4)^{\frac{1-n}{2}}$	$\frac{n-2}{2} > -\frac{1}{2}$
(32)	$J_{\frac{n-2}{2}}(a\lvert x\rvert)K_{\frac{n-2}{2}}(\beta\lvert x\rvert)$	$\frac{2^{3(\frac{n-2}{2})}(a\beta)^{\frac{n-2}{2}}\Gamma(\frac{n-1}{2})}{\sqrt{\pi}}.[(a^2+\beta^2+\lvert y\rvert^2)^2-4a^2\lvert y\rvert^2]^{\frac{1-n}{2}}$	$\frac{n-2}{2} > -\frac{1}{2}$, $Re\ \beta > \lvert Im\ a\rvert$
(33)	$(\lvert x\rvert^2+a^2)^{-\frac{n-1}{2}}$	$\frac{1}{2^{\frac{n}{2}}\pi^{-\frac{1}{2}}\Gamma(\frac{n+1}{2})a}\ e^{-a\lvert y\rvert}$	$\frac{n-1}{2} > -1$, $Re\ a > 0$

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(34)	$\delta(m^2+P)$	$\frac{2\pi^{n/2}}{2\Gamma(\frac{n}{2})}\frac{1}{(2\pi)^{n/2}}(m^2+Q)_+^{\frac{n-2}{2}}$
(35)	$\delta^{(k)}(m^2+P)$	$\frac{1}{2^{2k+\frac{n}{2}}\Gamma(\frac{n}{2}+k)}(m^2+Q)_+^{\frac{n-2}{2}+k}$
(36)	$\delta^{(k)}(P)$	$\frac{1}{2^{2k+\frac{n}{2}}\Gamma(\frac{n}{2}+k)}Q^{\frac{n-2}{2}+k}$
(37)	$\delta(P)$	$\frac{\Omega_n}{2}\frac{1}{(2\pi)^{n/2}}Q_+^{\frac{n-2}{2}}$ (*)
(38)	$\delta^{(k)}(m^2+P)*\delta^{(l)}(m^2+P)$	$C_{k,n}.C_{l,n}(m^2+Q)_+^{\frac{n-2}{2}+\frac{n-2}{2}+k+l}$, where $C_{s,n}=\frac{1}{2^{2s+\frac{n}{2}}\Gamma(\frac{n}{2}+s)}$, $s=0,1,\ldots$
(39)	$\frac{x_+^{\alpha-1}}{\Gamma(\alpha)}$	$\frac{s_+^{\frac{n}{2}-\alpha-1}}{2^{\frac{n}{2}-2\alpha}\Gamma(\frac{n}{2}-\alpha)}$
(40)	$\frac{x_-^{\alpha-1}}{\Gamma(\alpha)}$	$\frac{(-1)^{\frac{n}{2}}}{2^{\frac{n}{2}-2\alpha}}\frac{s_-^{\frac{n}{2}-\alpha-1}}{\Gamma(\frac{n}{2}-\alpha)}$

(*) $\Omega_n=\frac{2\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2})}$

Hankel Transforms

(41)	$\frac{\lvert x\rvert^{\alpha-1}}{\Gamma(\frac{\alpha}{2})}$	$\frac{\Gamma(\frac{\alpha+1}{2})}{2^{\frac{n}{2}-3\alpha+1}\pi^{1/2}}\{\frac{s_+^{\frac{n}{2}-\alpha-1}}{\Gamma(\frac{n}{2}-\alpha)}+\frac{(-1)^{n/2}s_-^{n/2-\alpha-1}}{\Gamma(\frac{n}{2}-\alpha)}\}$
(42)	$\frac{\lvert x\rvert^{\alpha-1}sgnx}{\Gamma(\frac{\alpha-1}{2})}$	$\frac{\Gamma(\frac{\alpha}{2})}{2^{\frac{n}{2}-3\alpha+1}\pi^{1/2}}[.\frac{s_+^{\frac{n}{2}-\alpha-1}}{\Gamma(\frac{n}{2}-\alpha)}-\frac{(-1)^{n/2}s_-^{n/2-\alpha-1}}{\Gamma(\frac{n}{2}-\alpha)}].$
(43)	$(x\pm i_0)^{\alpha-1}$	$\frac{\Gamma(\alpha)}{2^{\frac{n}{2}-2\alpha}\Gamma(\frac{n}{2}-\alpha)}(s-i_0)^{\frac{n}{2}-\alpha-1}$, if $\alpha\neq -k, k=0,1,\ldots$
(44)	$\frac{\lvert m^2+P\rvert^{\alpha-1}}{\Gamma(\frac{\alpha}{2})}$	$\frac{\Gamma(\frac{\alpha+1}{2})}{2^{\frac{n}{2}-3\alpha+1}\pi^{1/2}}\{\frac{(m^2+Q)_+^{\frac{n}{2}-\alpha-1}}{\Gamma(\frac{n}{2}-\alpha)}+(-1)^{n/2}\frac{(m^2+Q)_-^{\frac{n}{2}-\alpha-1}}{\Gamma(\frac{n}{2}-\alpha)}\}$
(45)	$\frac{\lvert m^2+P\rvert^{\alpha-1}sgn(m^2+P)}{\Gamma(\frac{\alpha+1}{2})}$	$\frac{\Gamma(\frac{\alpha}{2})}{2^{\frac{n}{2}-3\alpha+1}\pi^{1/2}}\{\frac{(m^2+Q)_+^{\frac{n}{2}-\alpha-1}}{\Gamma(\frac{n}{2}-\alpha)}-(-1)^{n/2}\frac{(m^2+Q)_-^{\frac{n}{2}-\alpha-1}}{\Gamma(\frac{n}{2}-\alpha)}\}$
(46)	$\delta^{(k)}(x)$	$\frac{1}{2^{\frac{n}{2}+2k}\Gamma(\frac{n}{2}+k)}s_+^{\frac{n-2}{2}+k}$
(47)	$\frac{(m^2+P)_+^{\alpha-1}}{\Gamma(\alpha)}$	$\frac{1}{2^{\frac{n}{2}-2\alpha}}\frac{(m^2+Q)_+^{\frac{n}{2}-\alpha-1}}{\Gamma(\frac{n}{2}-\alpha)}$
(48)	$\frac{(m^2+P)_-^{\alpha-1}}{\Gamma(\alpha)}$	$\frac{(-1)^{\frac{n}{2}}}{2^{\frac{n}{2}-2\alpha}}\frac{(m^2+Q)_-^{\frac{n}{2}-\alpha-1}}{\Gamma(\frac{n}{2}-\alpha)}$
(49)	$(m^2+P\pm i0)^{\alpha-1}$	$\frac{\Gamma(\alpha)}{2^{\frac{n}{2}-2\alpha}\Gamma(\frac{n}{2}-\alpha)}(m^2+Q\mp i_0)^{\frac{n}{2}-\alpha-1}$, $\alpha\neq -k$, $k=0,1,\ldots,n$ even
(50)	$\delta^{(m)}$	$\frac{\Gamma(\frac{n+2m-1}{2})s^{\frac{n-2}{2}+m}}{2^{2m+\frac{n-2}{2}+1}\Gamma(\frac{n-2}{2}+m+\frac{1}{2})\Gamma(\frac{n+2m}{2})}$
(51)	$\delta_a^{(m)}$	$\frac{1}{2^{m+1}}\frac{J_{\frac{n-2}{2}+m}(\sqrt{as})}{(\sqrt{as})^{\frac{n-2}{2}+m}}s^{\frac{n-2}{2}+m}$
(52)	δ	$\frac{1}{2^n}\frac{1}{\Gamma(\frac{n}{2})}s^{\frac{n-2}{2}}$

Chapter III Laplace Transforms

	$f(x)$	$\mathcal{L}[f(t)] = \int_{R^n} e^{-i<t,z>}\Phi(t)dt$
(1)	$\frac{x_+^{\alpha-1}}{\Gamma(\alpha)}$	$e^{-i\frac{\pi}{2}}(y-i0)^{-\alpha}$
(2)	$W(t,\alpha,m,n) =$ $= \begin{cases} \frac{(m^{-2}u)^{\frac{\alpha-n}{4}} J_{\frac{\alpha-n}{2}}\{(m^2u)^{1/2}\}}{\pi^{\frac{n-2}{2}} 2^{\frac{\alpha+n-2}{2}} \Gamma(\frac{\alpha}{2})} & \text{if } t \in \Gamma_+ \\ 0 & \text{if } t \notin \Gamma_+ \end{cases}$.	$(\rho^2+m^2)^{-\frac{\alpha}{2}}$, $m \in R^+$, $\rho^2 = z_1^2 + \cdots + z_{n-1}^2 - z_0^2$.
(3)	$W(t,\alpha,m=0,n) =$ $R_\alpha(u) =$ $= \begin{cases} \frac{u^{\frac{\alpha-n}{2}}}{H_n(\alpha)} & \text{if } t \in \Gamma_+ \\ 0 & \text{if } t \notin \Gamma_+ . \end{cases}$.	$(\rho^2)^{-\frac{\alpha}{2}}$ $H_n(\alpha) = \pi^{\frac{n-2}{2}} 2^{\alpha-1}\Gamma(\frac{\alpha}{2})\Gamma(\frac{\alpha-n+2}{2})$, $\alpha \in C$, n is the dimension of the space.
(4)	$G_R(t,m^2,\alpha,n) =$ $= \frac{(u-m^2)_+^{\alpha-1}}{\Gamma(\alpha)} =$ $= \begin{cases} \frac{(u-m^2)^{\alpha-1}}{\Gamma(\alpha)} & \text{if } u-m^2>0 \text{ and } t_0>0 \\ 0 & \text{if } t \text{ belongs to the complementary set.} \end{cases}$.	$2^\alpha(2\pi)^{\frac{n-2}{2}} m^{\alpha+\frac{n-2}{2}} \rho^{-\alpha+\frac{2-n}{2}} . K_{\alpha+\frac{n-2}{2}}(m\rho)$
(5)	$G_R(t,m=0,\alpha,n)$	$(2\pi)^{\frac{n-2}{2}} 2^{2\alpha+n/2-2} \rho^{-2\alpha+2-n}\Gamma(\alpha+\frac{n-2}{2})$
(6)	$G_R(t,m^2,\alpha=1,n)$	$2(2\pi)^{\frac{n-2}{2}} m^{n/2}\rho^{-n/2}K_{n/2}(m\rho)$
(7)	$G_R(t,m=0,\alpha=1,n)$	$2^{n/2}(2\pi)^{\frac{n-2}{2}}\rho^{-n/2}\Gamma(\frac{1}{2})$

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(8)	$G_R(t, m^2, \alpha = -k, n) = \delta_R^{(k)}(u = m^2)$	$2^{-k}(2\pi)^{\frac{n-2}{2}} m^{-k+\frac{n-2}{2}} \rho^{k+\frac{2-n}{2}} K_{-k+\frac{n-2}{2}}(m\rho)$
(9)	$\delta_R^{(k)}(u - m^2)$	$\frac{(2\pi)^{\frac{n-2}{2}}}{(z_1^2+\cdots+z_{n-1}^2-z_0^2)^{\frac{n-2}{4}}}(-1)^k D^k_{\lambda=m^2} K_{\frac{n-2}{2}}[\lambda^{1/2}(z_1^2 + \cdots + z_{n-1}^2 - z_0^2)^{1/2}]$
(10)	$G_R(t, m = 0, \alpha = -k, n) = \delta_R^{(k)}(u)$	$(2\pi)^{\frac{n-2}{2}} 2^{-2k+\frac{n}{2}-2}\Gamma(-k + \frac{n-2}{2})\rho^{-n+2k+2}$, valid if $-k + \frac{n-2}{2} \neq 0, 1, \ldots$
(11)	$G_R(t, m^2, \alpha = 0, n) = \delta_R(u - m^2)$	$(2\pi)^{\frac{n-2}{2}} m^{\frac{n-2}{2}} \rho^{\frac{2-n}{2}} K_{\frac{n-2}{2}}(m\rho)$, valid if $k \neq 0, m^2 > 0$ or $k = 0, m = 0$.
(12)	$G_R(t, m = 0, \alpha = 0, n) = \delta_R(u)$	$2^{n/2}(2\pi)^{\frac{n-2}{2}}\Gamma(\frac{n-2}{2})\rho^{2-n}$
(13)	$R_0(u) = \delta(x)$	1
(14)	$W_0(u, m) = \delta(x)$	1
(15)	$(s^2)^{\frac{\alpha-n}{2}}$ $s = \sqrt{t_0^2 - t_1^2 - \cdots - t_{n-1}^2}$	$(\rho^2)^{-\frac{\alpha}{2}}$

(16)	$\delta^{(k)}(x)$	$z^k (Re\ z \geq 0)$
(17)	$H(x)$	$\frac{1}{z}$
(18)	$H(x) \log x$	$\frac{-\log z - C}{z}$ C =Euler's constant. $C = lim_{n\to\infty}(1 + \frac{1}{2} + \cdots + \frac{1}{n} - \log n)$
(19)	$Pf \frac{H(x)}{x^k}$	$\frac{(-1)^k}{(k-1)!} z^{k-1} (\log z - C - \sum_{e=1}^{k-1} \frac{1}{e})$ C = Euler's constant
(20)	$x^k e^{\alpha x} H(x) \quad (\alpha \in C,\ Re\ z > \ Re\ \alpha)$	$\frac{k!}{(z-\alpha)^{k+1}}; \quad Re\ z > \ Re\ \alpha$
(21)	$e^{-x} H(x)$	$\frac{1}{z+1}; \quad Re\ z > -1$
(22)	$x^\beta H(x) \quad (\beta \neq -1, -2, \ldots)$	$\frac{\beta!}{z^{\beta+1}}$

Laplace Transforms

(23)	$H(x)\cos\beta x$	$\frac{z}{z^2+\beta^2}$	$(Re\ z > \|Im\ \beta\|)$
(24)	$H(x)\sin\beta x$	$\frac{\beta}{z^2+\beta^2}$	$(Re\ z > \|Im\ \beta\|)$
(25)	$H(x)\cosh\beta x$	$\frac{z}{z^2-\beta^2}$	$(Re\ z > \|Im\ \beta\|)$
(26)	$H(x)\sinh\beta x$	$\frac{\beta}{z^2-\beta^2}$	$(Re\ z > \|Im\ \beta\|)$
(27)	$H(x)\|\sin ax\| \quad [a>0]$	$\frac{a\coth(\frac{\pi z}{2a})}{z^2+a^2}$	$(Re\ z > 0)$
(28)	$x^{\beta}H(x)\ln x \quad [\beta \neq -1,-2,\ldots]$	$\frac{\beta!}{z^{\beta+1}}\{\psi(\beta)-\ln z\}$	$(Re\ z > 0)$
(29)	$x^{\nu}I_{\nu}(\beta x)$	$\frac{(\nu-\frac{1}{2})!(2\beta)^{\nu}}{(z^2+\beta^2)^{\nu+\frac{1}{2}}\pi^{\frac{1}{2}}}$ $(Re\ z > \|Im\beta\|,\ Re\ \nu > -\frac{1}{2})$	
(30)	$x^{\frac{\nu}{2}}I_{\nu}(2x^{\frac{1}{2}}$	$z^{-\nu-1}e^{\frac{1}{z}}$	$(Re\ z > 0,\ Re\ \nu > -1)$

Glossary of Symbols

3	R^n
3	$r = \|x\|^2$
3	$P = P(x)$
3	$(P + i0)^{\lambda}$
3	$Q = Q(y)$
3	$T(P \pm i0, \lambda)$
3	$\tilde{T}$
3	S_{R+}
3	S'_{R+}
3	$S^{\sharp}_{R^n}$
4	$\rho = \|y\|^2$
4	$\hat{f}(x)$
5	C
5	$G_{\alpha}(P \pm i0, m, n)$
5	$K_{\nu}(z)$
5	$I_{\nu}(z)$
5	$H_{\alpha}(m, n)$
5	$H_{\alpha}(P \pm i0, n)$
5	K^l
5	L^k
6	$\triangle^s$
6	$H_{\alpha}(\|x\|^2, n)$
6	$R_{\alpha}(x, n)$
7	$H^1_{\nu}(z)$
7	$H^2_{\nu}(z)$
7	$J_{\nu}(z)$
7	$Y_{\nu}(z)$
7	C^n

7 Γ_+

7 Γ_-

7 T_+

7 T_-

8 $F[\phi]$

8 $L[\phi]$

10 δ

10 $W(t, \alpha, m^2, n)$

11 Ω_n

11 $\Psi(z)$

11 $(\mathcal{H}\{\phi(t)\})$

12 $\mathcal{H}(U(t))$

15 x_+^λ

15 x_-^λ

15 $|x|^\lambda$

15 $|x|^{\alpha-1} sgn\ x$

15 $\Gamma(\alpha)$

15 $\frac{x_+^{\alpha-1}}{\Gamma(\alpha)}$

15 $\frac{x_-^{\alpha-1}}{\Gamma(\alpha)}$

15 $\frac{|x|^{\alpha-1}}{\Gamma(\frac{\alpha}{2})}$

15 $\frac{|x|^{\alpha-1}\ sgn\ x}{\Gamma(\frac{\alpha+1}{2})}$

16 $P_\nu^\mu(z)$

16 ${}_mF_n$

16 L_k^α

16 $L_k^0(z)$ (Laguerre polynomial)

16 $M_{k,\mu}$ (Whittaker's functions)

16 ${}_1F_1(a, c, z)$ (Kummer's confluent hypergeometric series)

17 P_+^λ

17 P_-^λ

18	$(m^2 + P \pm i0)^\lambda$
18	$(m^2 + Q)_+^\lambda$
18	$(m^2 + Q)_-^\lambda$
19	$(m^2 + P \pm i0)^{-k}$
19	$T(\|x\|^2, \lambda)$
19	Pf
25	$u_1^\beta = \|u\|^\beta H(u^2)$
25	$u_2^\beta = \|u\|^\beta H(u^2) H(x_1)$
26	$u_3^\beta = \|u\|^\beta H(-u^2)$
26	$u_4^\beta = \|u\|^\beta H(u^2) H(-x_1)$
26	$\|u\| = \|u^2\|^{1/2}$
27	$u^2 = x_1^2 - x_2^2 - \cdots - x_n^2, \qquad n > 2$
30	$u^2 = x_1^2 + \cdots + x_p^2 - x_n^2$

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